

DATA ASSIMILATION PRACTICAL I

Effect of statistical analysis parameters

written by*

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Introduction

These are a set of matlab programs to show the results of analysing a small set of idealised observations, given a (one-dimensional) background field, and assumed background and observation error covariances. The aim is to allow you to explore the effect of varying the statistical parameters of simple data assimilation systems.

In each case, information from the observations, \mathbf{y} , is combined with information from a background field, \mathbf{x}_b , to produce an analysis field, \mathbf{x}_a , according to the standard OI (Optimal Interpolation) expression

$$\mathbf{x}_a = \mathbf{x}_b + \mathbf{B}\mathbf{H}^T (\mathbf{H}\mathbf{B}\mathbf{H}^T + \mathbf{R})^{-1} (\mathbf{y} - \mathbf{H}\mathbf{x}_b). \quad (1)$$

\mathbf{B} and \mathbf{R} are the background and observation error covariance matrices, respectively, and \mathbf{H} is the observation operator (in these examples, it is just a simple linear interpolation). Note that these matlab routines explicitly invert the matrix, even though it is not usually the most efficient way of solving the equation.

The elements of the background error covariance matrix \mathbf{B} are given by

$$\mathbf{B}_{ij} = \sigma^2 \rho_{ij}, \quad (2)$$

where σ is the error standard deviation. The ρ_{ij} are given by a second-order autoregressive correlation function of the distance r_{ij} between the grid-points i and j

$$\rho_{ij} = \left(1 + \frac{r_{ij}}{L}\right) \exp\left(\frac{-r_{ij}}{L}\right), \quad (3)$$

where L is the correlation length scale. A similar expression is used for the elements of \mathbf{R} , where the observation error correlations (if any) are assumed to depend on the distance between observations.

Each routine is run by typing its name at the matlab command line. If you answer ‘Yes’ to the question ‘Use default parameter values?’, the program will run with its default values. If you answer ‘No’, you will be asked various questions, such as where you want to put the observations, and the error characteristics of the observations and background field. Note that if you want to change an array, you will need to enter the correct number of new values.

The routine will plot a graph showing the observations, background and analysis, which you will be given the option of saving as a ‘.fig’ file. Finally, a summary of the selected analysis parameters is printed to the screen.

*based on original notes by R. Swinbank & R. Petrie, DARC, UK

Exercise 1: analysis_2obs

This program calculates an analysis from two observations using the OI formula (1). A list of inputs into the program and their default values is given in Table 1 below.

variable	description	default value
o1	value of observation 1	1.0
x1	x position of observation 1	4.0
o2	value of observation 2	-1.0
x2	x position of observation 2	6.0
sigo	observation error standard deviation, σ_o	1.0
Lo	observation error correlation length scale	0.0
sigf	background error standard deviation, σ_f	1.0
Lf	background error correlation length scale	2.0

Table 1: default parameter values for exercise 1

In this exercise the background field is assumed to be zero everywhere. The observations should take values between -2 and +2 and their horizontal (x axis) positions must be between 0 and 10.

Figure 1 shows the output when the default parameter values in Table 1 are used. The dotted lines are the analyses that would have been produced if just one of the observations had been available and the dashed line is the analysis that is produced when both observations are used.

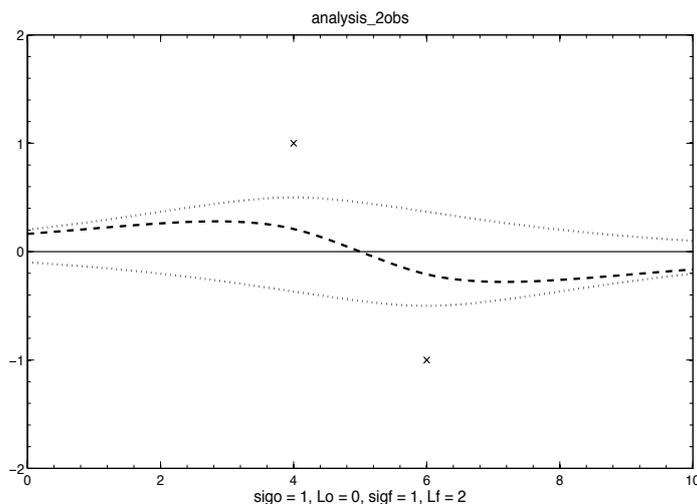


Figure 1: analysis_2obs with default parameters from Table 1.

Try to reproduce this figure by calling the script `'analysis_2obs'` from the matlab command line and selecting `'yes'` when asked if you want to use the default parameter values.

Now explore what happens if you change the inputs:

- what happens if you change the observation values and locations?
- what happens if you vary the background error correlation length? (e.g. from 0 to 10)
- what happens if you vary the background error standard deviation? try both increasing and decreasing.

The effect of the observation error covariances is less obvious:

- what happens if the observation errors are highly correlated and the observations agree with one another?
- what happens if they disagree with one another?

Try to explain the results you find.

Exercise 2: analysis_mobs

This program calculates an analysis of two sets of observations, A and B, with a maximum of seven in each set. As in the previous exercise, the observations should take values between -2 and +2 and their horizontal (x axis) positions must lie between 0 and 10. The default set up is as follows:

SET A: 3 observations

position	4.0	5.0	6.0
value	1.0	1.0	1.0

observation error standard deviation, $\text{sigma}_a = 2.0$
 observation error length scale, $\text{Lo}_a = 0.0$

SET B: 3 observations

position	4.0	5.0	6.0
value	-1.0	-1.0	-1.0

observation error standard deviation, $\text{sigma}_b = 1.0$
 observation error length scale, $\text{Lo}_b = 2.0$

The background field is assumed to be zero everywhere and has default error standard deviation, $\text{sig}_f = 1.0$, and error lengthscale, $\text{L}_f = 1.0$.

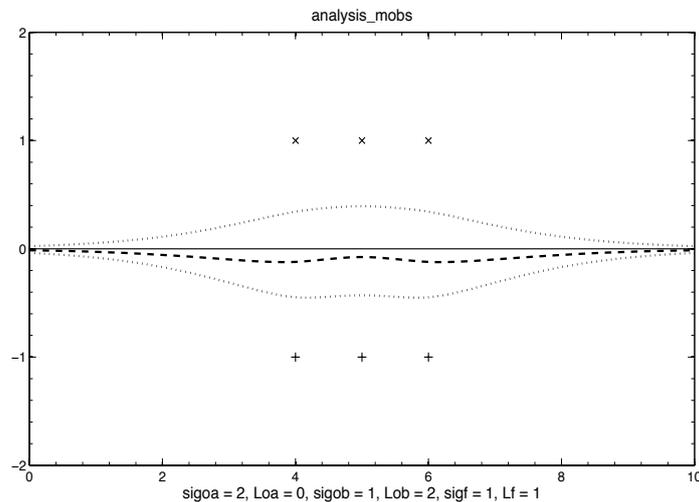


Figure 2: output from *analysis_mobs* with default parameters. The dotted lines are the analyses that are produced if only one set of observations is used, the dashed line is the analysis that is produced when both sets of observations are used.

Again, explore what happens if you change the inputs. For example,

- examine the effect of varying the number of observations, their error magnitudes, and their spatial density.
- examine the effect of varying the observation error correlation lengthscales.

how do these modifications relate to each other?

- try using one set of observations to represent sparse but accurate observations and the other set to represent dense but inaccurate observations?

Try to explain the results you find.

Exercise 3: analysis_sim

This program calculates an analysis using a background field and observations generated from a given 'truth' field.

You are given 3 options for defining the truth:

1. zero everywhere
2. psuedo step function
3. sum of several sinusoidal curves

The background field is generated either by adding random perturbations to the truth (according to the specified background error characteristics), or by shifting the truth field along the x -axis. The default background parameters are: error standard deviation, $\text{sigf} = 2.0$, and lengthscale, $\text{Lf} = 2.0$.

The observations are generated by adding random perturbations to the truth using the specified observation error statistics. There can be up to 11 observations, with horizontal (x axis) positions between 0 and 10. The default is 5 observations at positions [3.0, 4.0, 5.0, 6.0 & 7.0] with observation error standard deviation, $\text{sigo} = 1.0$, and error lengthscale, $\text{Lo} = 0.0$.

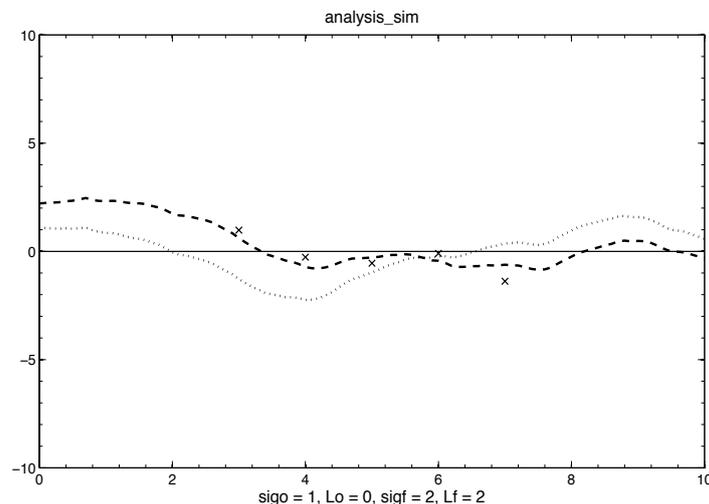


Figure 3: output from *analysis_sim* with truth equal to zero everywhere, background = truth + random perturbation, and default background and observation parameters. The observations are given by crosses, the dotted line is the background field and the dashed line is the analysis.

Once again, explore what happens if you change the inputs.

- try experimenting with different truth functions.
- try varying the number of observations, their errors and lengthscales with different background settings.
- an interesting exercise is to use the ‘step function’ form of the truth field with the background field equal to the truth, but shifted by a couple of units; which parameter settings are best for getting the analysis to move the step back to the correct location?

Further exercises

The matlab routines used in these exercises are quite basic. Feel free to take copies and amend the code to enable you to carry out your own explorations. For example, you could easily amend *analysis.sim* to explore what happens if the assumed observation and background errors are different from their true values.