Gravity
and its role in earth sciences

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Lecture Two

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Lecture Two:

Objective of this lecture

is to address the following three questions:

1. What can we learn from the earth‘s gravitational field about the physics of earth system?
2. What are the underlying principles?
3. What are the current state-of-the-art and main challenges?

Remember from Lecture One:

Gravitation at any chosen point

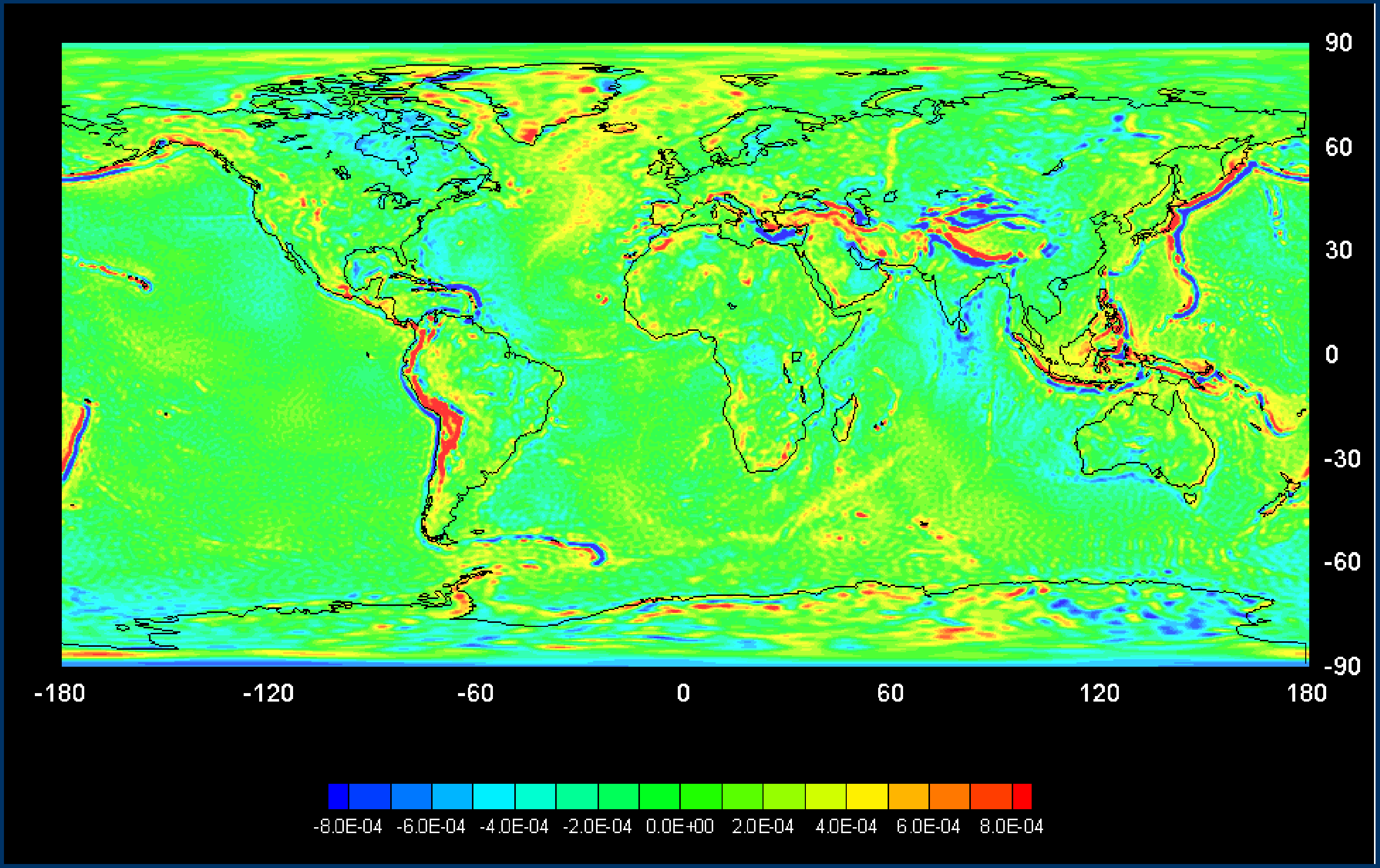
is the integral attraction of all matter of the universe:

- primarily of the (almost) solid earth
- also of ice, oceans and atmosphere (changing),
- and of moon, sun and planets (moving)

Measurement of the global earth gravity field opens three areas of application:

1. A look into the earth's interior: state of mass (im-)balance and geodynamics
2. The geoid as a reference (level) of sea level, global ocean circulation and height systems
3. Variations of gravity (and geoid) as a measure of mass exchange processes in earth system

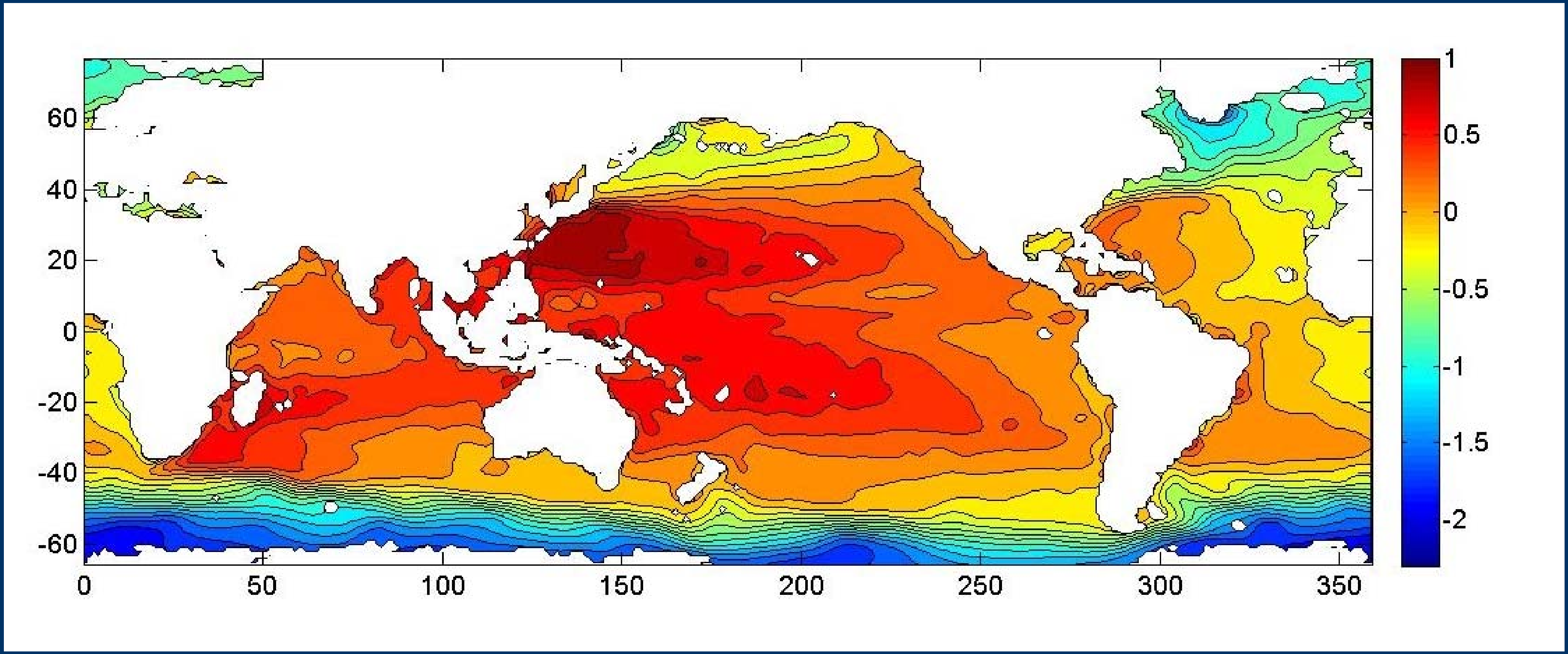
Theme 1: a look into the earth‘s interior



world map showing gravity anomalies

based on two months of GOCE data

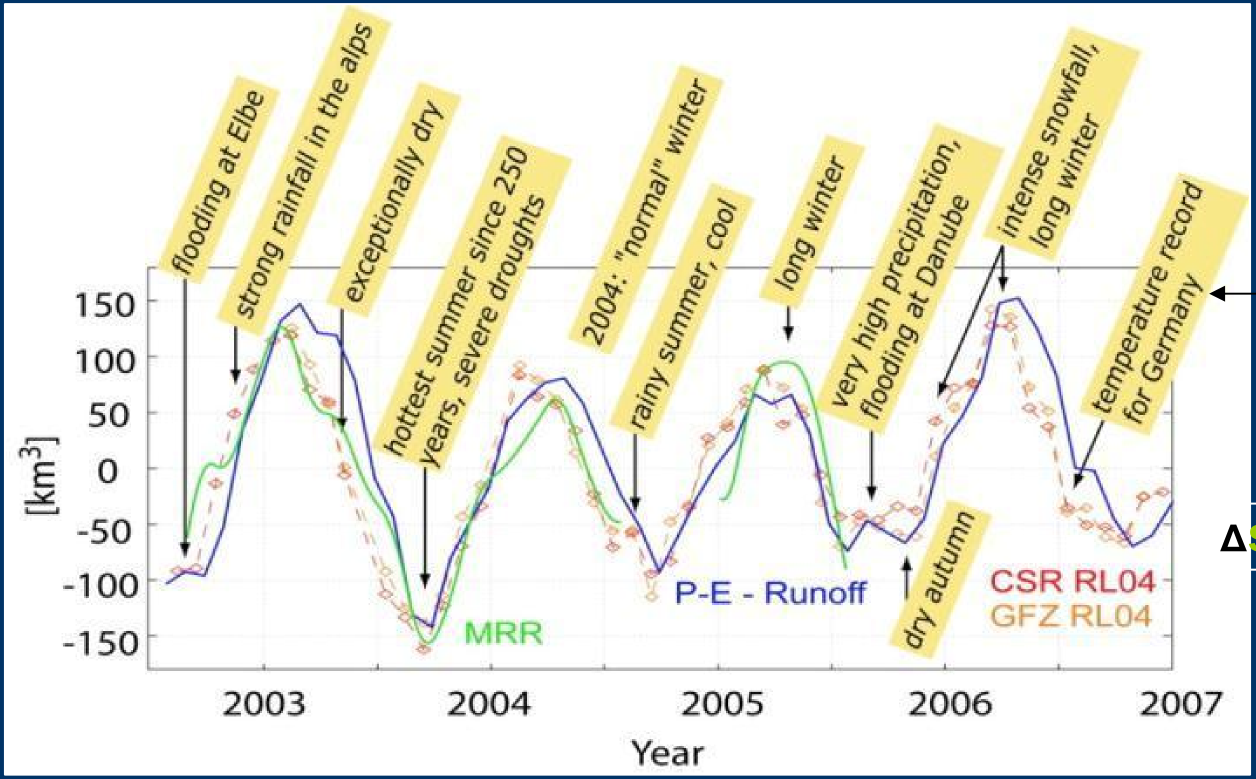
Theme 2: geoid as a reference to ocean topography



world map showing dynamic ocean topography

derived from satellite altimetry and GOCE

Theme 3: temporal variations of gravity



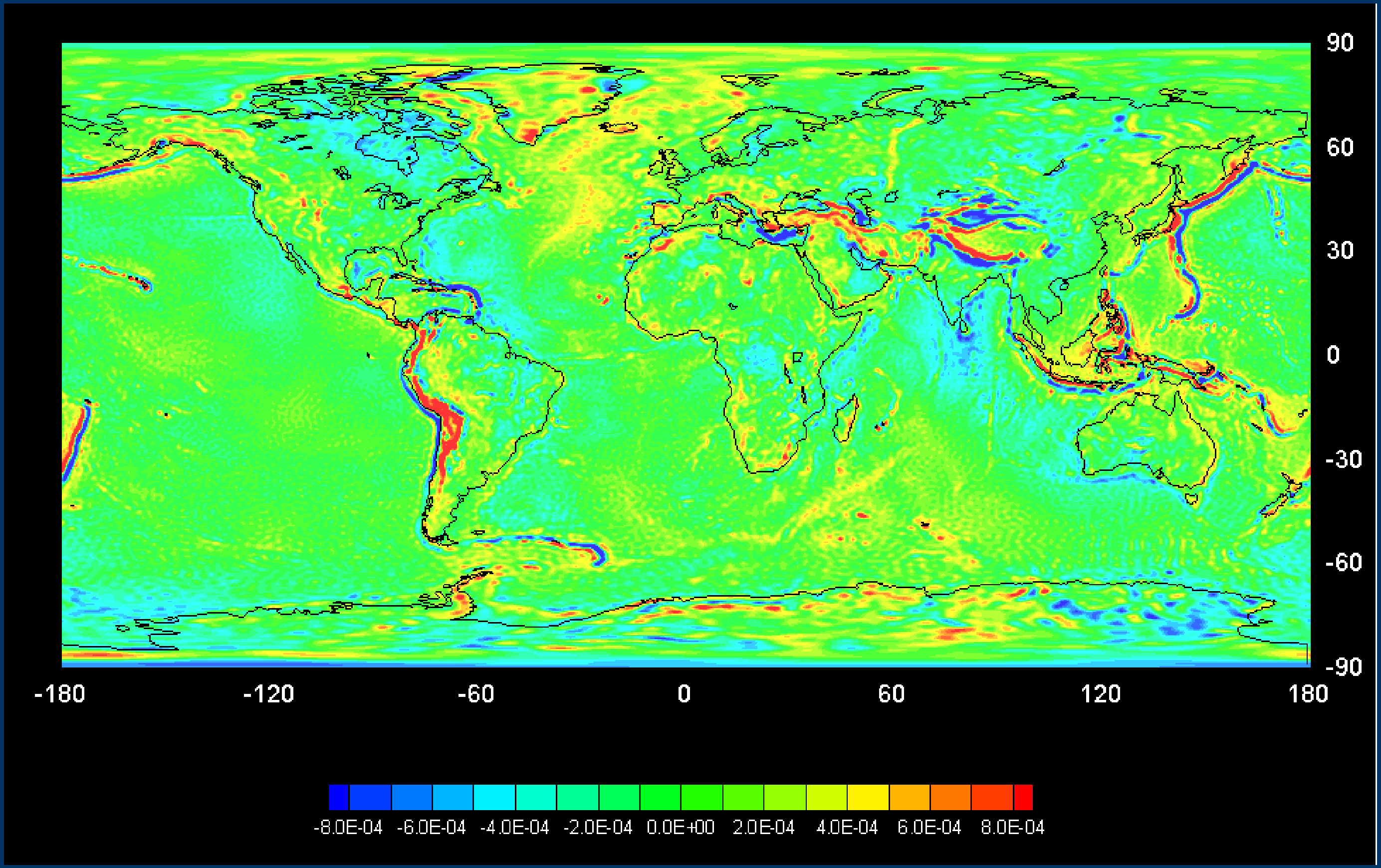
ΔS(GRACE)

Seitz et al., EPSL,2008

(sub-)seasonal water storage variability in Central Europe (from GRACE)

from gravity to geodynamics

Theme 1: a look into the earth‘s interior



world map showing gravity anomalies

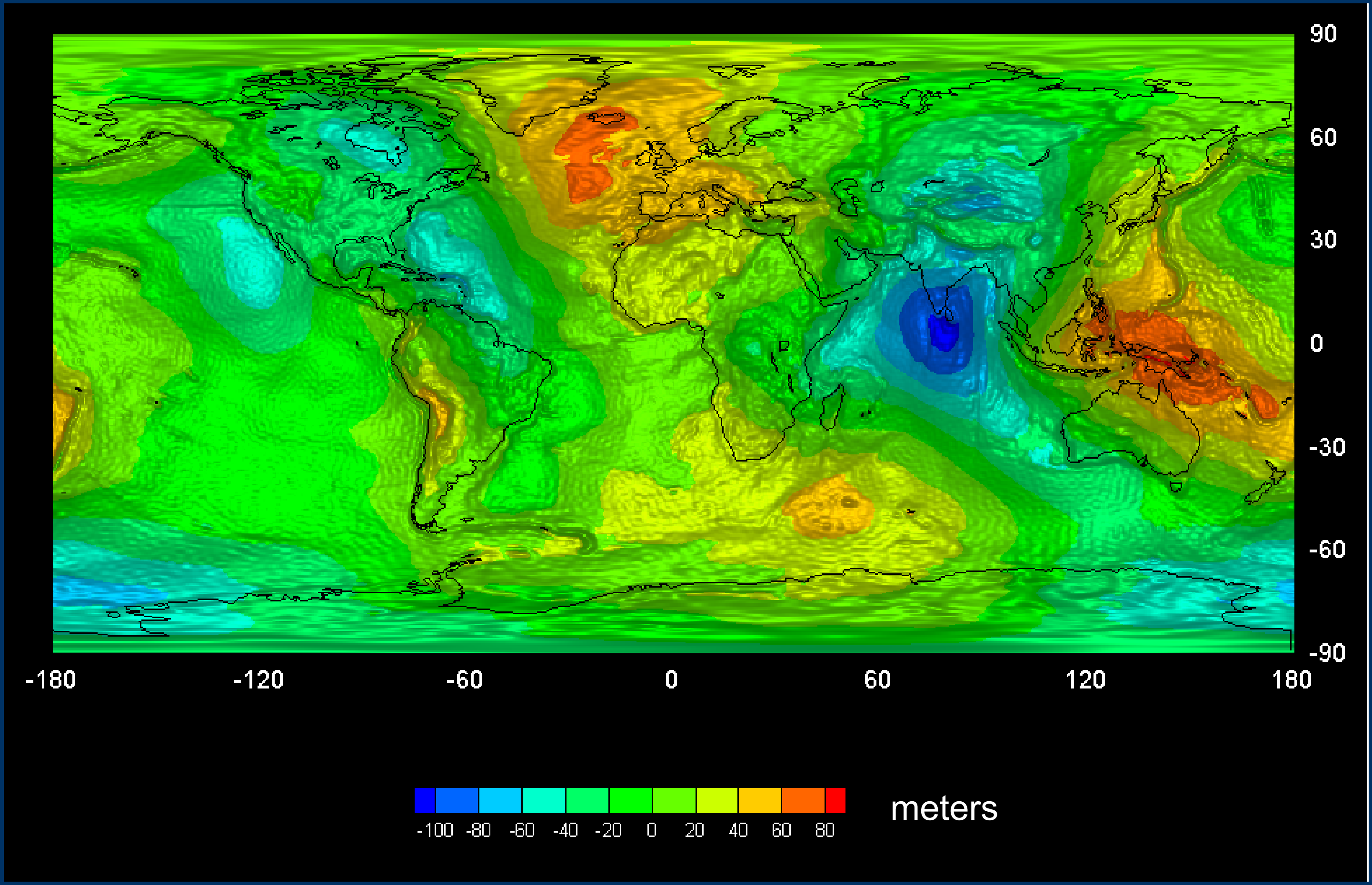
based on two months of GOCE data

from gravity to geodynamics

"Observations of the gravitational field of the Earth
thus provides a null experiment,
where the net result is
a small number determined by the difference of large effects"
[Richards & Hager, 1984]

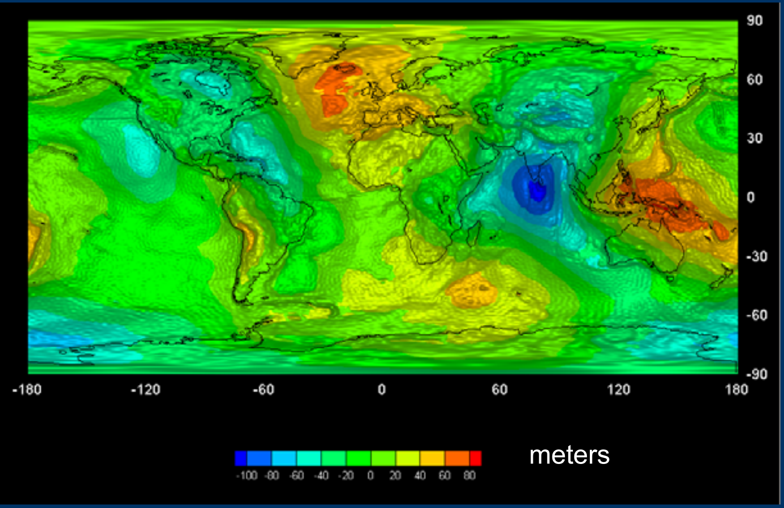
from gravity to geodynamics

a global geoid map based on two months of GOCE data



What do we see at large scales and at short scales?

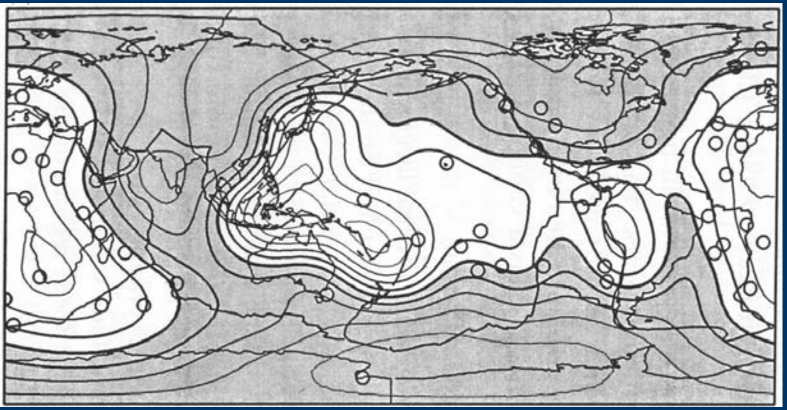
from gravity to geodynamics



What do we see at large scales ?

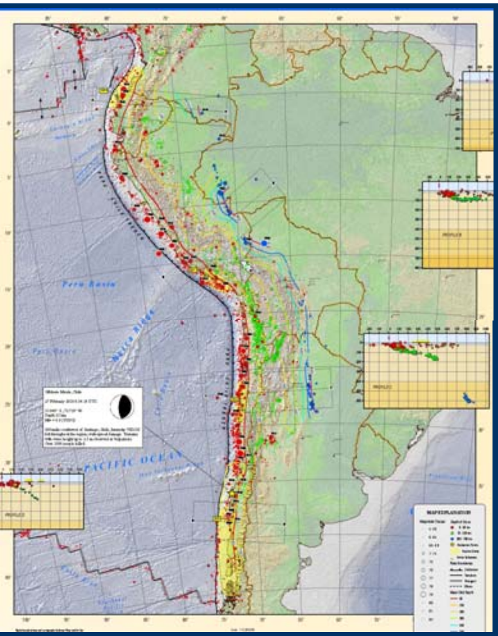
- little resemblance to topography and tectonic plates
- geoid highs at convergence zones and concentrations of hot spots
- only at convergence zones association with topography/ plates
- primary source of large scales: deep mantle convection

Richards & Hager, 1988

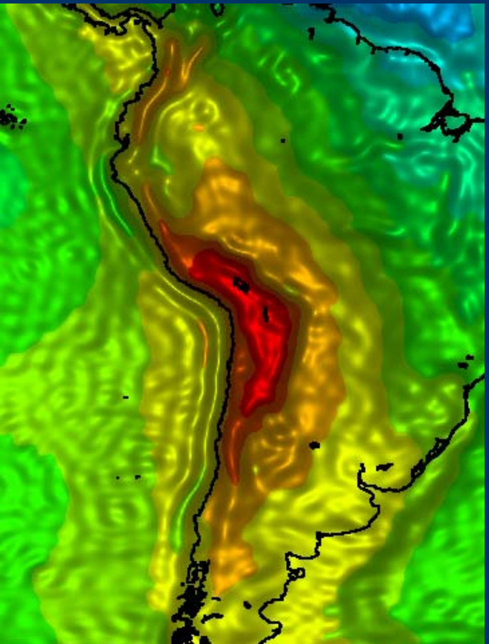


Hager and Richards, 1989

from gravity to geodynamics



USGS: Seismicity
of the Earth 1900-2007
Nazca Plate and South America



What do we see at short scales ?

- at first sight gravity anomalies resemble topographic heights
- a closer look reveals: gravity anomalies as derived from topography show marked differences to the observed ones
- these differences are a measure of mass balance (= isostasy)
- various concepts of isostasy exist, i.e. of mechanisms of compensation of topographic loads
- classical: Airy, Pratt, Vening-Meinesz modern: flexure of the lithosphere and mantle viscosity, thermal

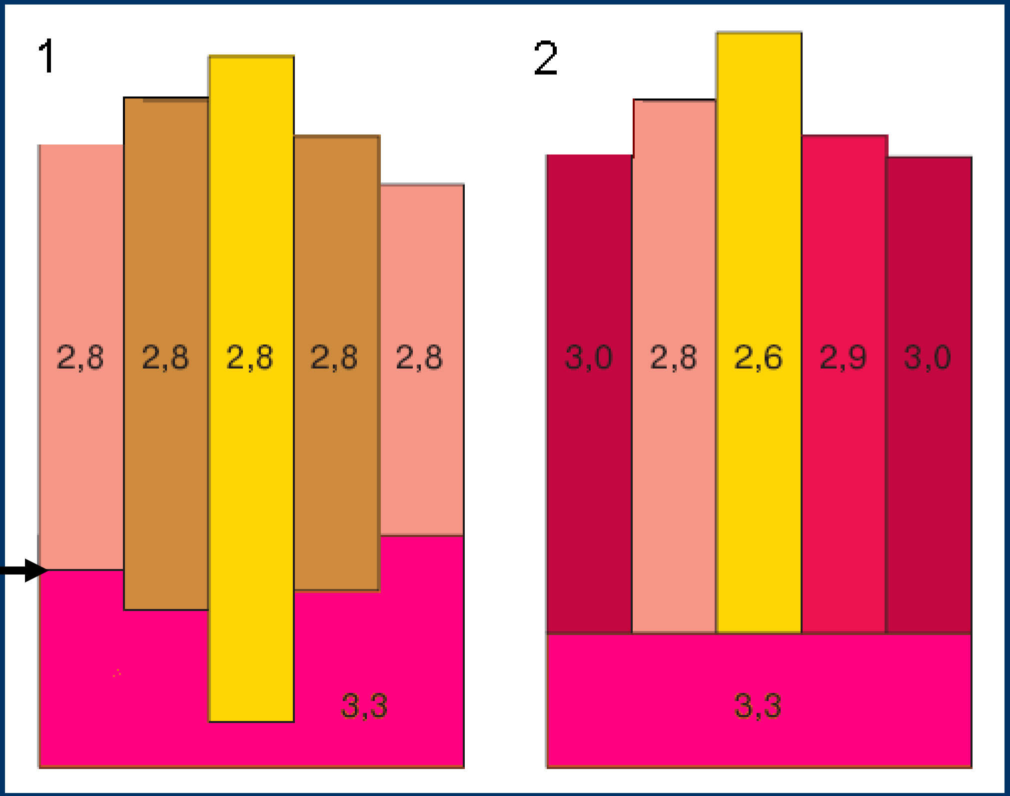
Fowler CMR, 2008; Turcotte & Schubert, 2002; Watts, 2001

from gravity to geodynamics

classical concepts of isostasy:

Airy (left) and Pratt (right)

Moho discontinuity



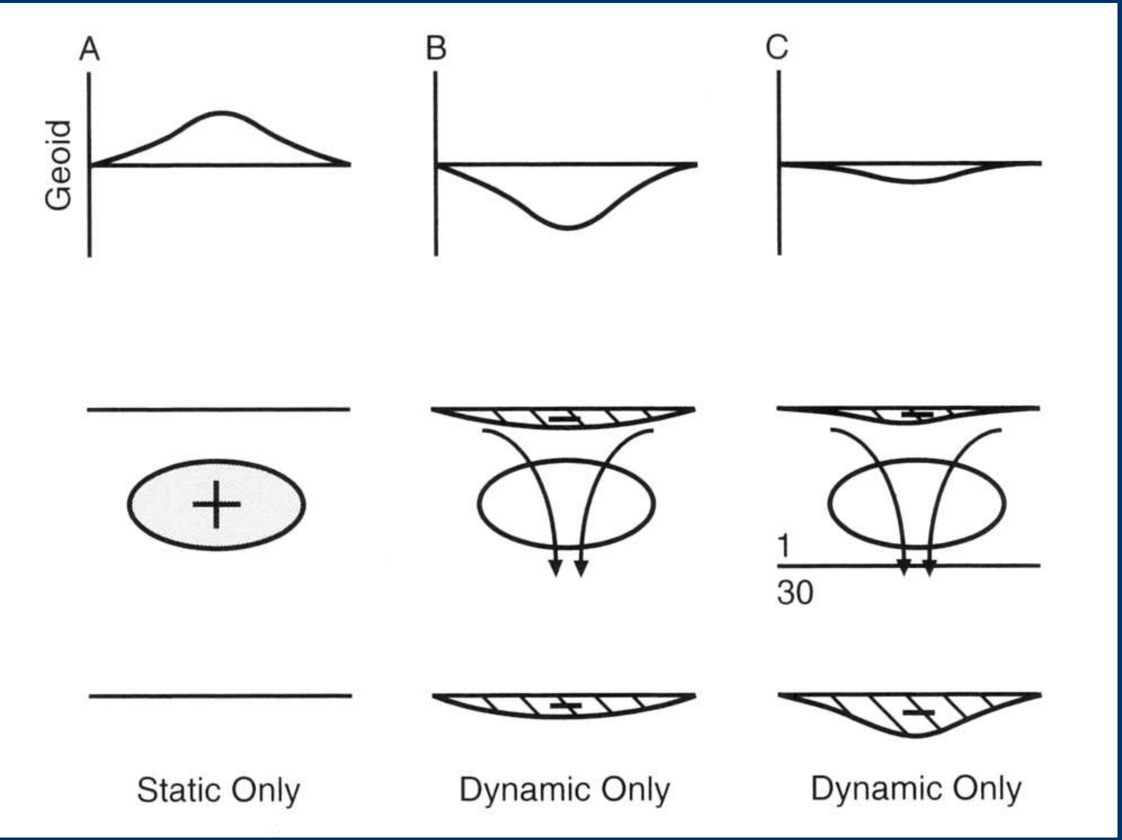
Source:wikipedia

the crust„swimming“ in the upper mantle forms an anti-root

lithospheric columns of equal weight on top of the astenosphere

from gravity to geodynamics

the superposition of two fundamental counteracting effects
(A) dynamic topography and (B) density contrast



Schubert, Tucotte & Olson, 2001

gravitational field

Newton's law

$$V\left(\vec{x}\right)=G\iiint_{v}\frac{\rho\left(\vec{x}'\right)}{\left|\vec{x}-\vec{x}'\right|}dv$$

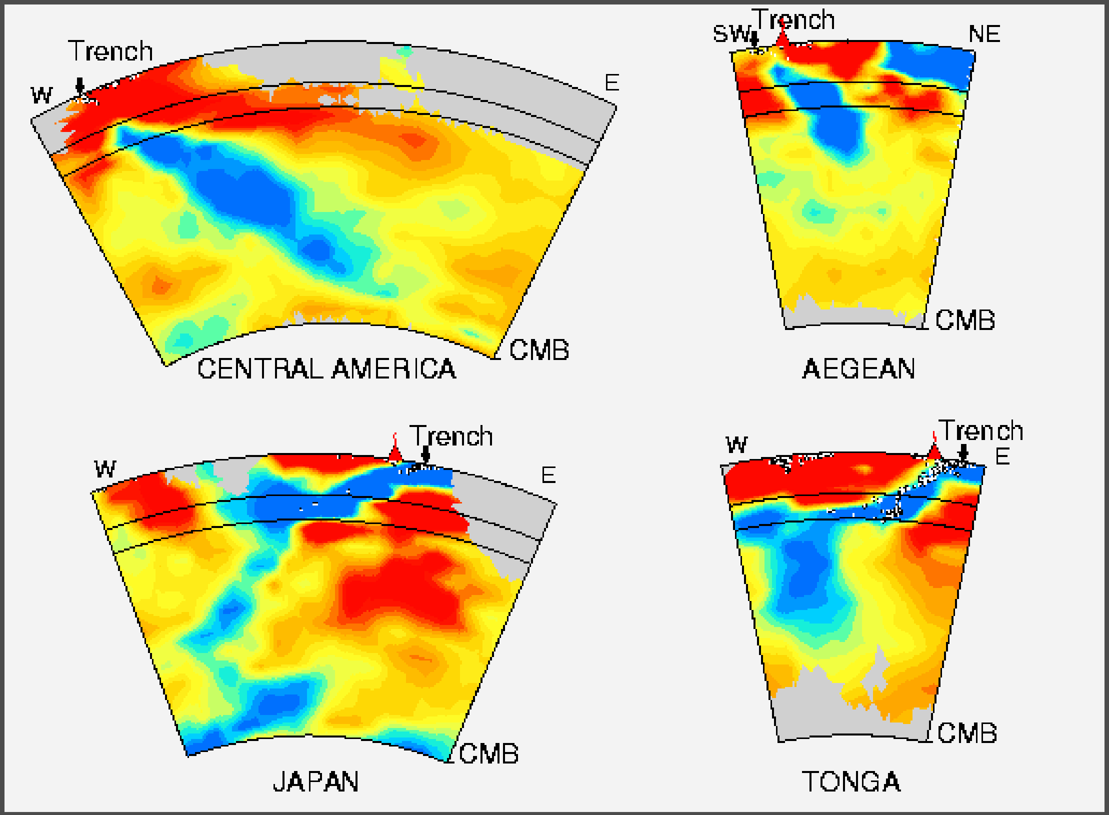
G = Newton's gravitational constant

discussion:

- integral over all masses
- density as an intrinsic property of all masses
- effect of outer masses well known: luni-solar tides
- gravity almost stationary
- time variations due to earthquakes, GIA and plate tectonics
- gravity is a vector field almost radial, everywhere
- however: inverse problem

solid Earth observation

most prominent observation technique in solid earth studies: seismic tomography

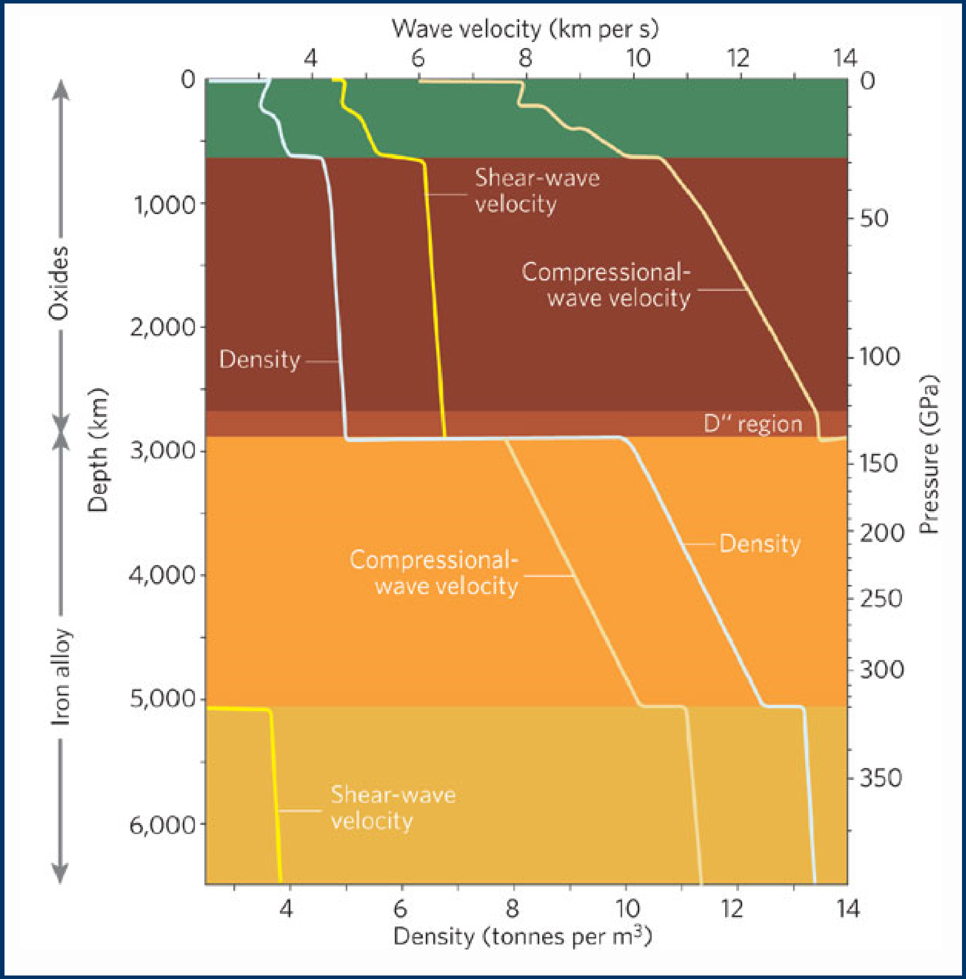


van der Hilst, Grand, Masters, Trampert, 2004 (?)

blue = high seismic velocity   red = low seismic velocity

solid Earth observation

velocity of shear waves and compressional waves as well as density as a function of depth

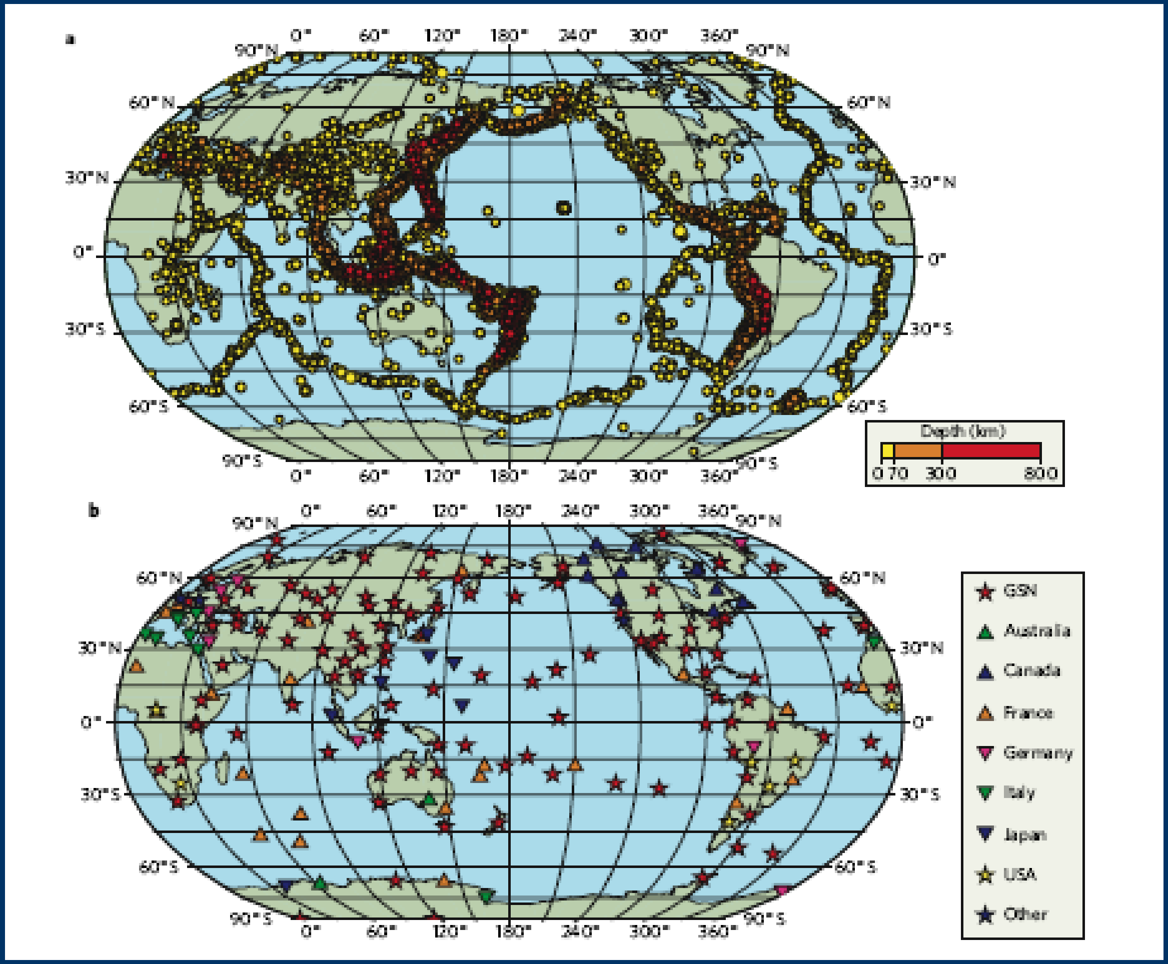


Romanowicz, nature, 2008

from gravity to geodynamics

signal source are earthquakes (at plate boundaries)

measured with seismometers in a global network (mostly on land)



Romanowicz, nature, 2008

seismic tomography:

- sources: earthquakes (at plate boundaries)
- seismometers: global networks (mostly on land)
- output of inversion:

  3D images of shear wave velocities v_s

  and/or of compressional wave velocities v_p

discussion:

  inversion is not unbiased

  translation of the v_s and v_p to density

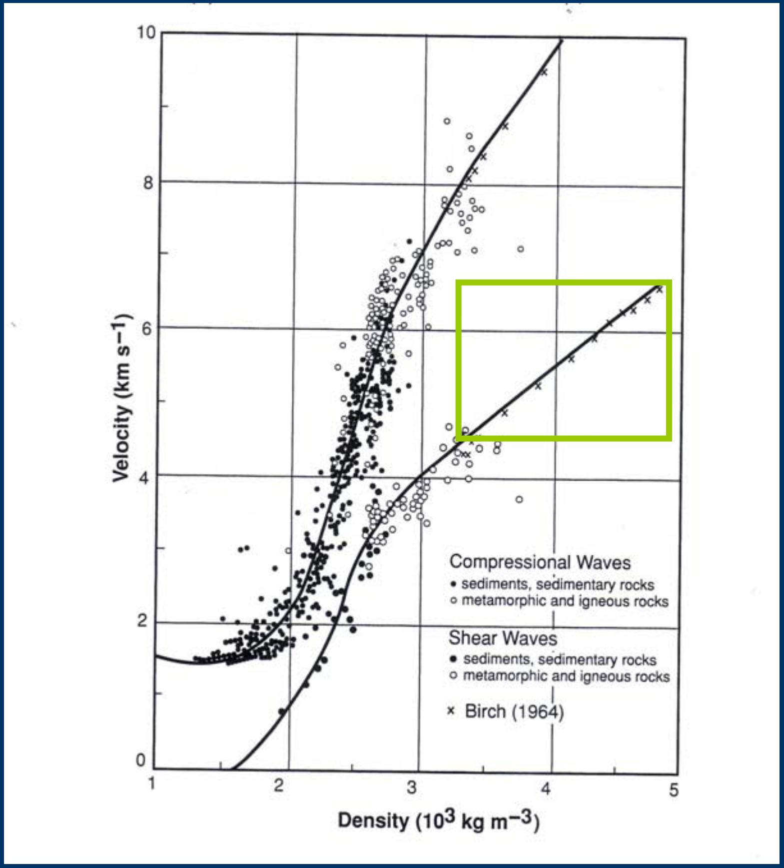
  depends also on shear modulus and bulk modulus

  i.e. it is non unique

from gravity to geodynamics

the difficulty of converting seismic velocities to density:

the answer: joint inversion with gravity and geoid



Fowler, 2008

$$\alpha = v_p = \sqrt{\frac{K + \frac{4}{3}\mu}{\rho}}$$

$$\beta = v_s = \sqrt{\frac{\mu}{\rho}}$$

$\rho = density$,

$\mu = shear$ mod$ulus$

$K = bulk$ mod$ulus$

Birch's law:

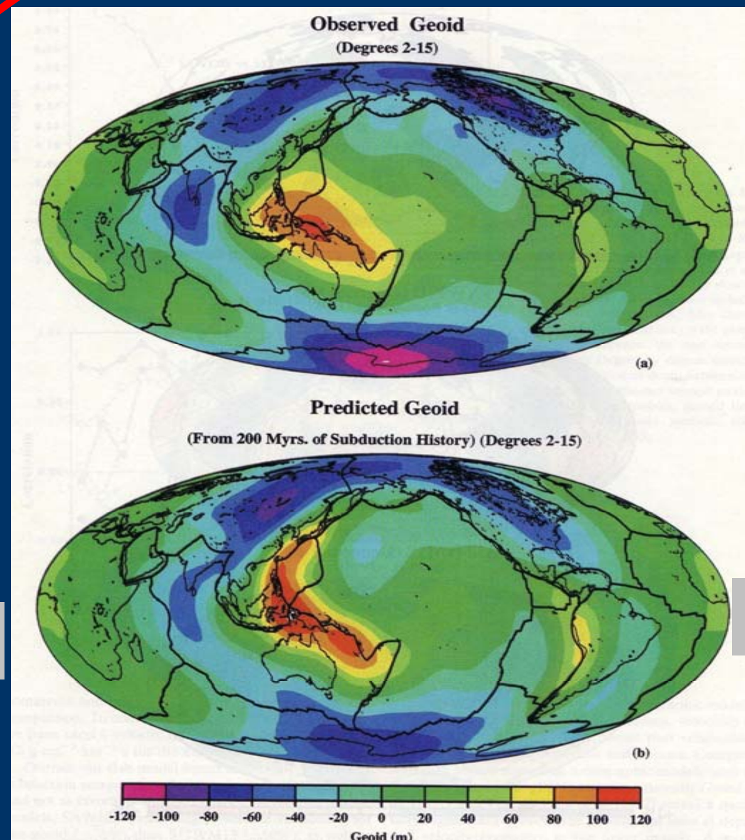
$$v = a\rho + b$$

from gravity to geodynamics

seismology

gravity and geoid

magnetic field



topography

crustal motion

from: Lithgow-Bertelloni & Richards, 1998 in Rev. of Geophysics

laboratory research

(pressure, temperature, composition)

basic equations of mantle convection

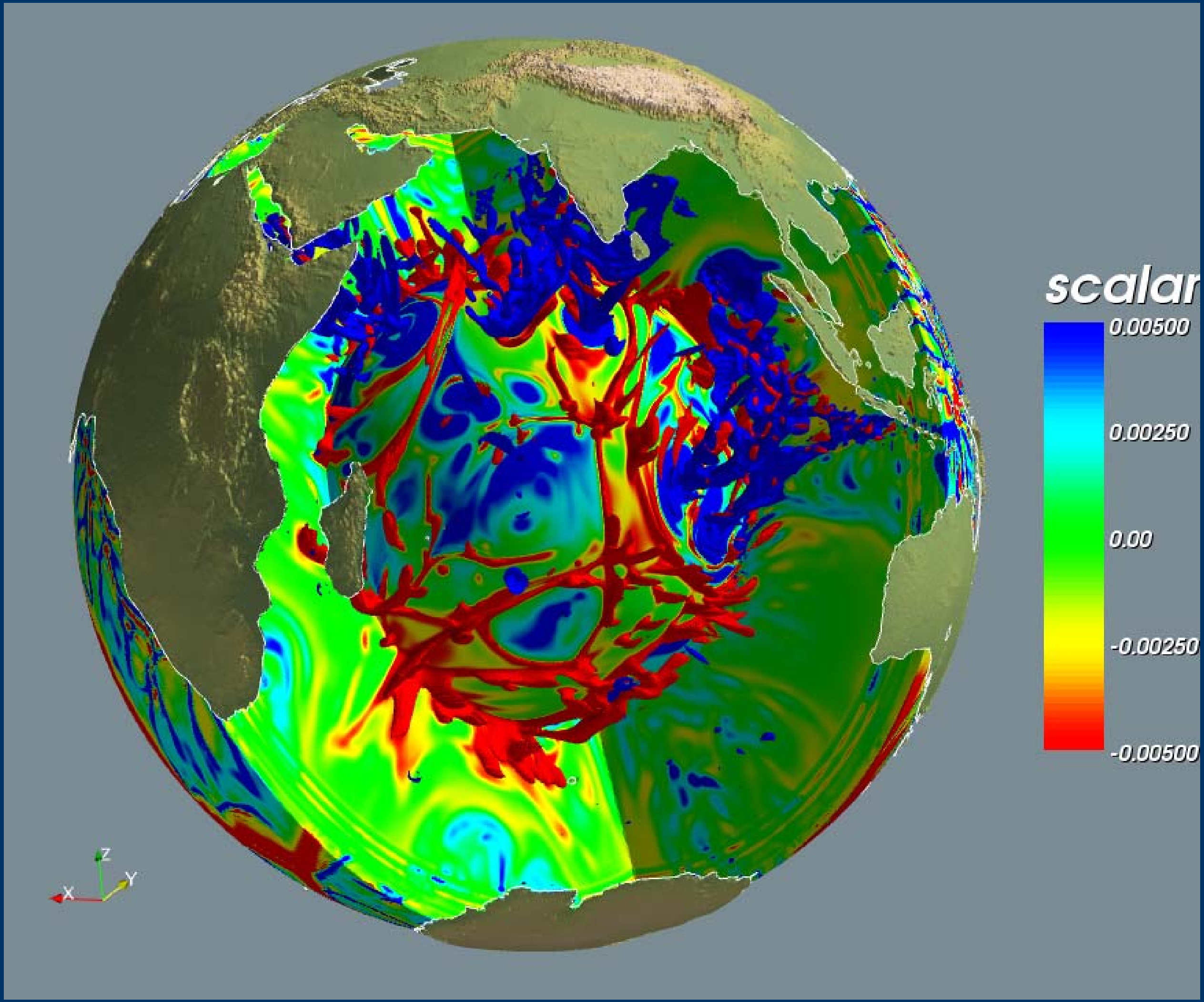
$$0 = \nabla \cdot \mathbf{u}$$

$$0 = -\nabla p + \nabla \cdot (\nu \nabla \mathbf{u}) + R(\bar{T} - T)\hat{\mathbf{k}}$$

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \nabla^2 T + h$$

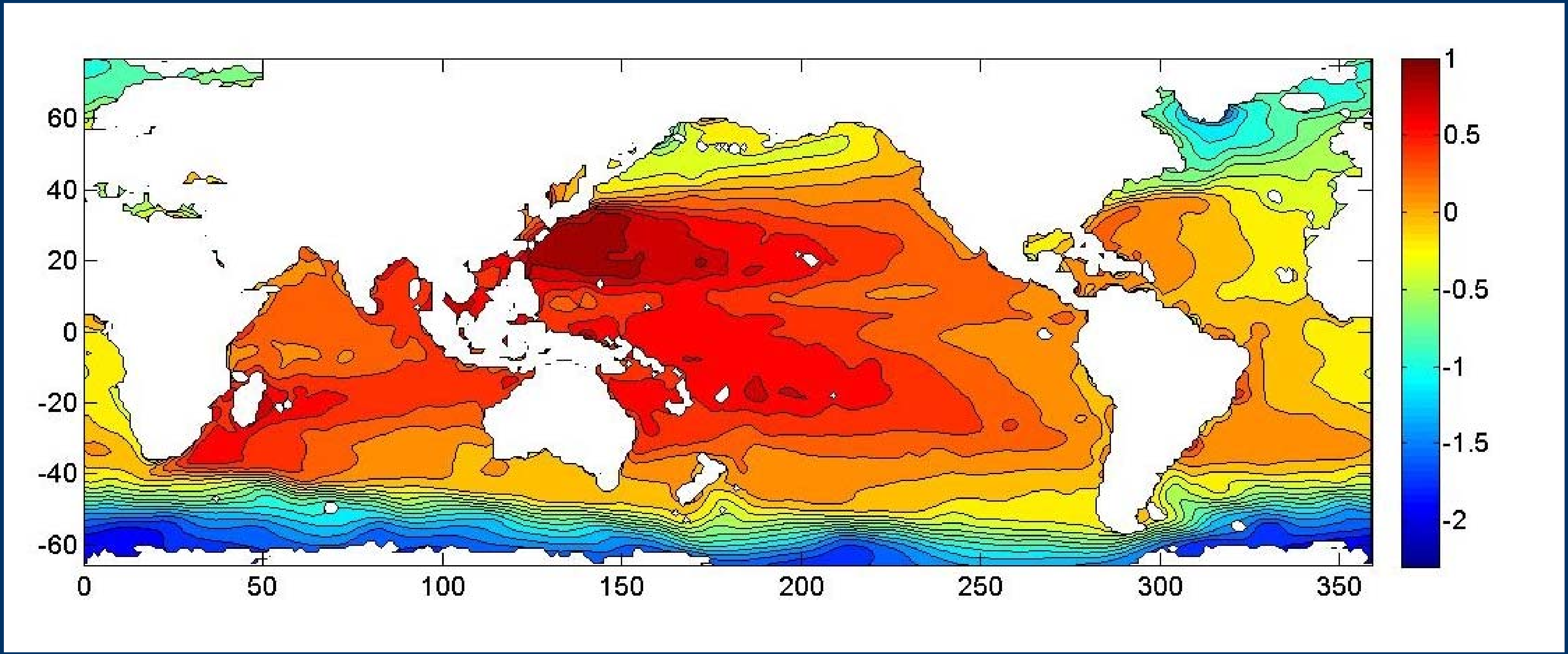
conservation of mass, linear momentum and energy

global geodynamic Earth model



source: H-P Bunge

Theme 2: geoid as a reference surface to ocean topography



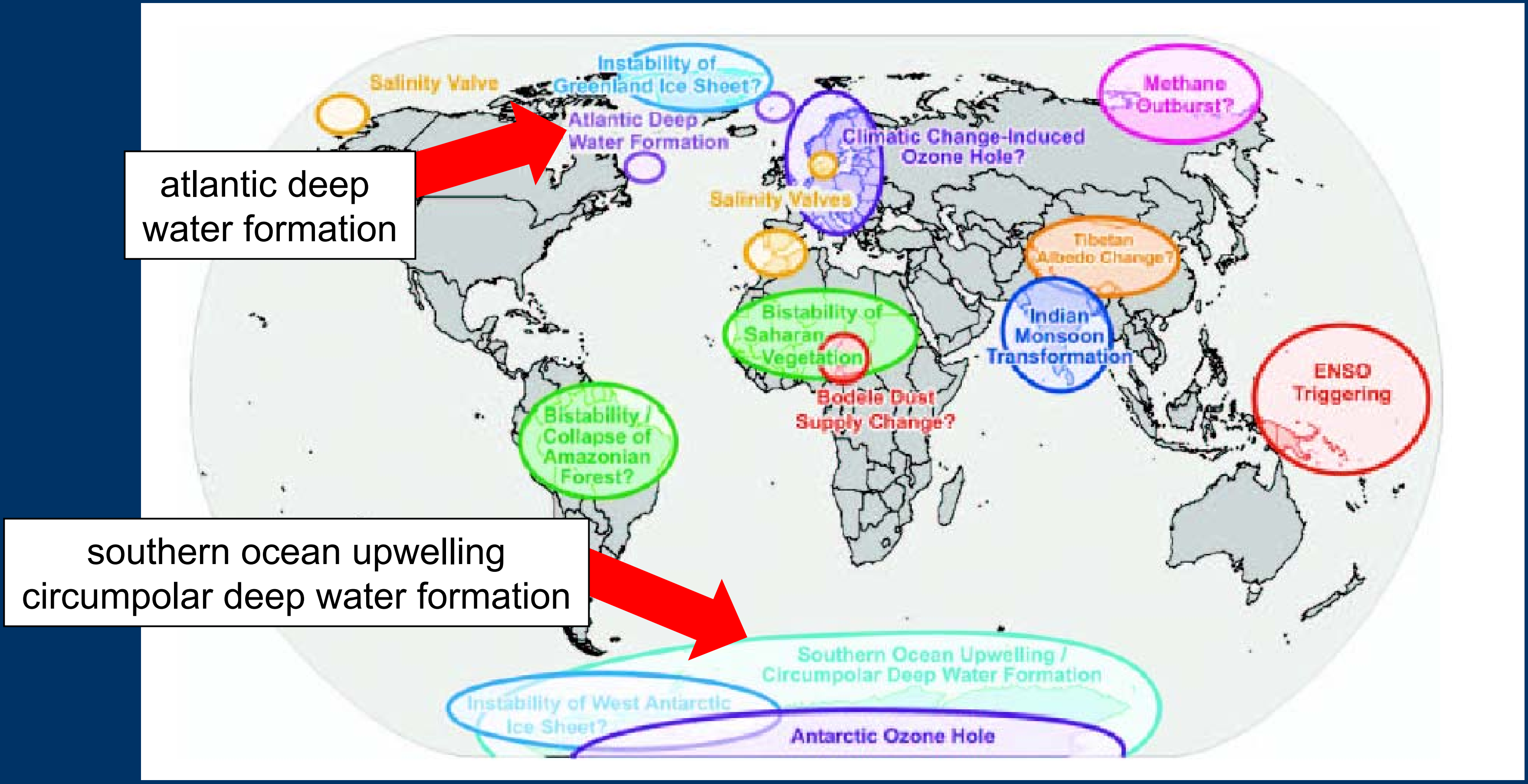
world map showing dynamic ocean topography

derived from satellite altimetry and GOCE

gravity as a global reference

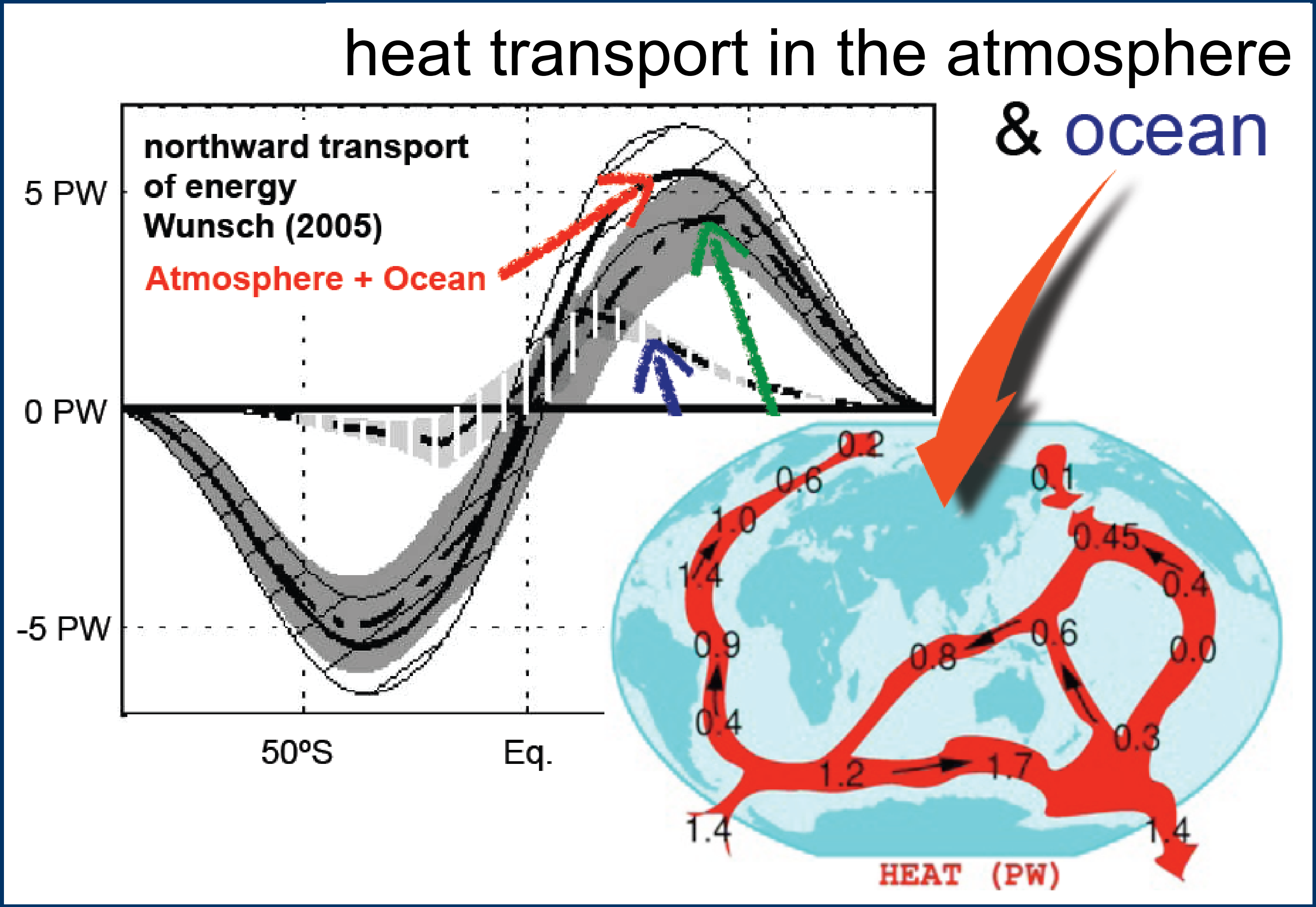
"The rate for ocean storage (of CO_2) is obtained not by measurement
but by subtracting large and uncertain numbers
pertaining to the atmosphere and biosphere.
This uncertainty is intolerable.
All we know for sure is
that the oceans are an important sink of heat and CO_2 and of ignorance."
[A. Baggeroer, W. Munk, Sept. 1992]

„tipping points“ of climate system



[nature,437, p.1238, 2005]

gravity as a global reference

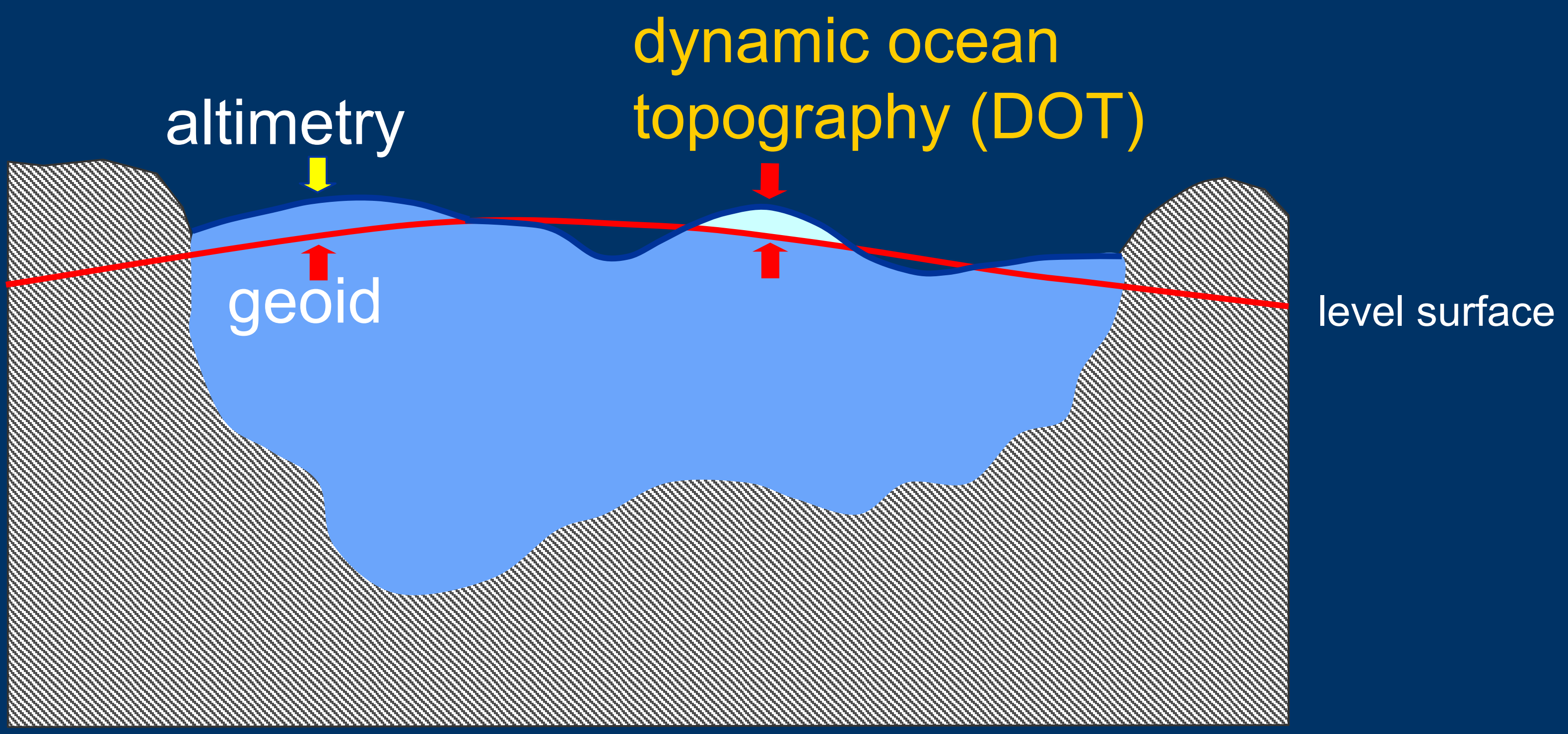


Losch, 2010

heat transport from equator region polewards:

is it 50% (textbooks) or 20 to 30%?

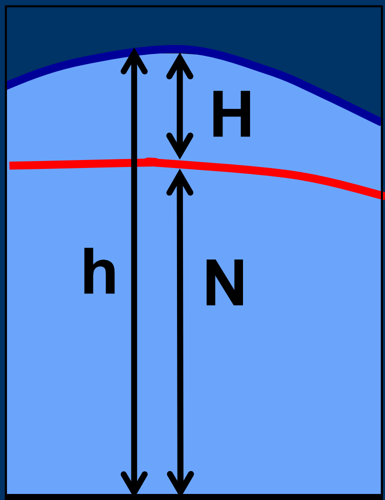
gravity as a global reference



dynamic ocean topography (DOT) or mean dynamic topography (MDT): deviation of the actual mean ocean surface from the geoid (hypothetical surface of ocean at rest); size 1 to 2 m; surface circulation follows contour lines of DOT

principle

gravity as a global reference



h
from altimetry

T/P, JASON 1&2,
ERS-1,2, ENVISAT,…

N
from gravity model

new generation missions:
- CHAMP
- GRACE
- **GOCE**

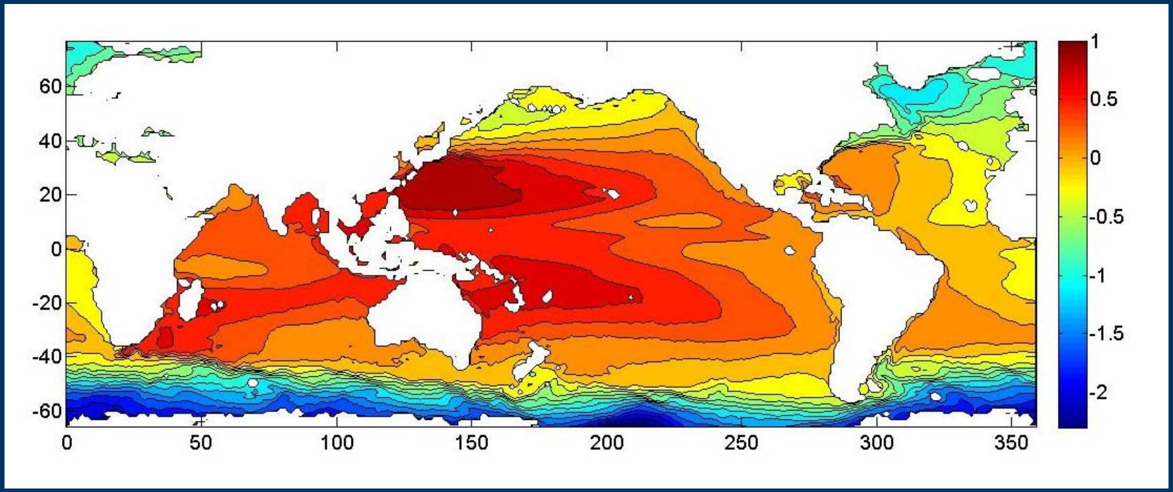
H
from ocean model

- SST
- WOCE hydrography
- XBT's
- moorings
- salinity/temperature profiles
- drifter velocities
- wind stress fields
- ARGOS
- …

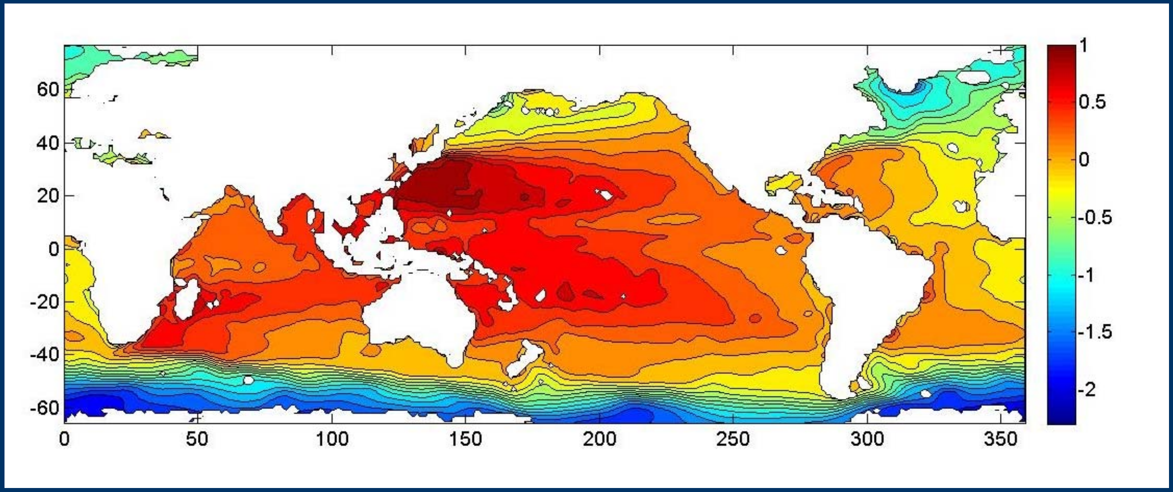
(Wunsch & Stammer, 2003)

principle

gravity as a global reference



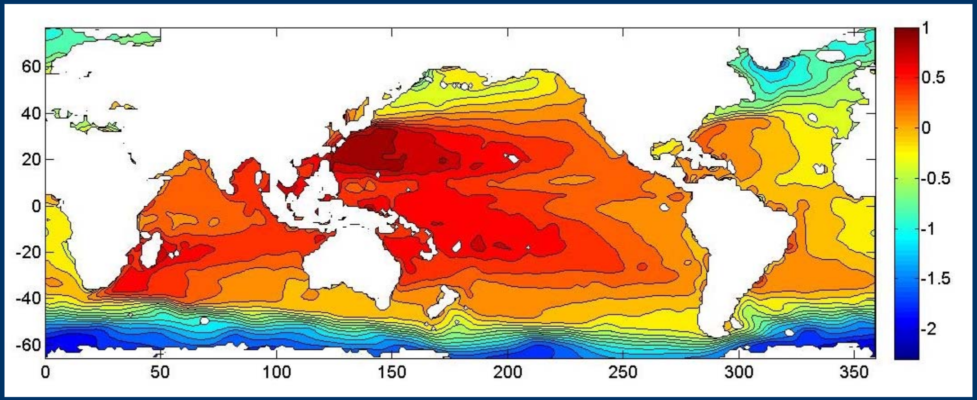
Maximenko et al., 2009



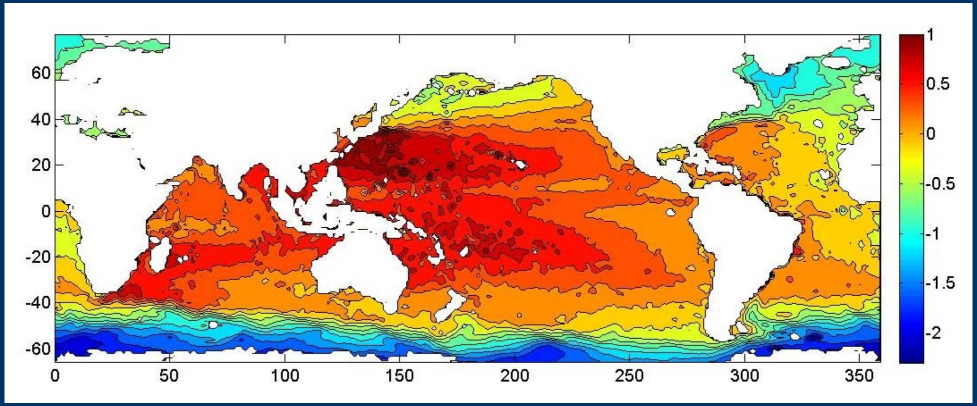
from GOCE and altimetry

mean dynamic ocean topography

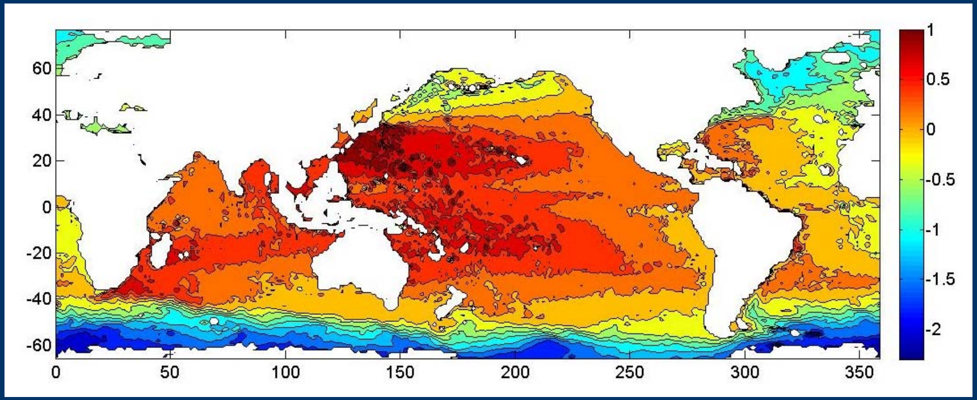
gravity as a global reference



low resolution

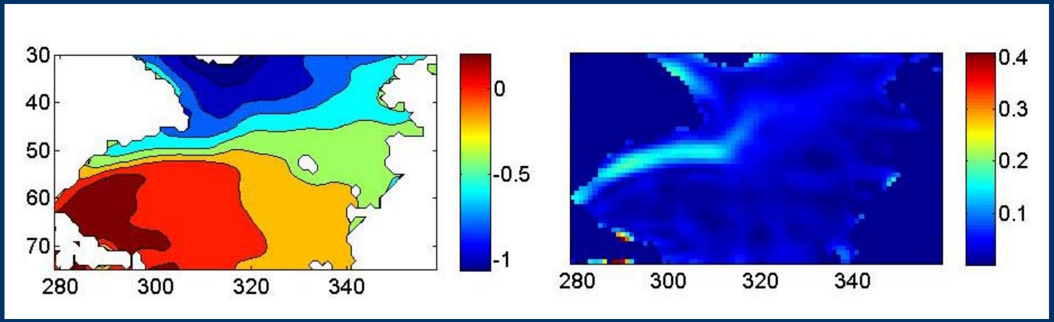


medium resolution

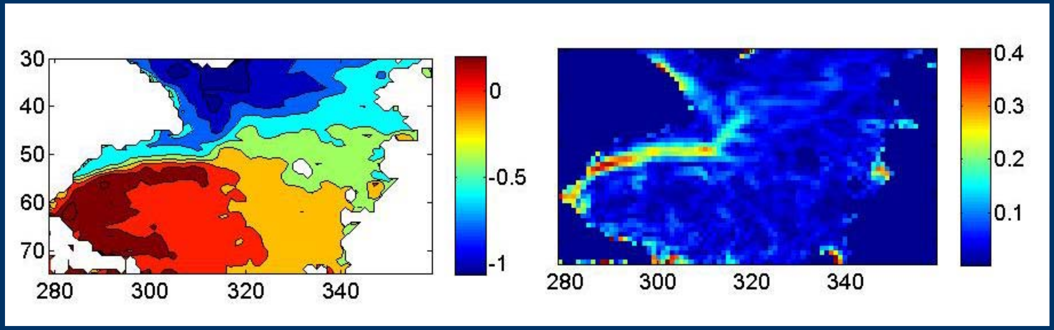


high resolution

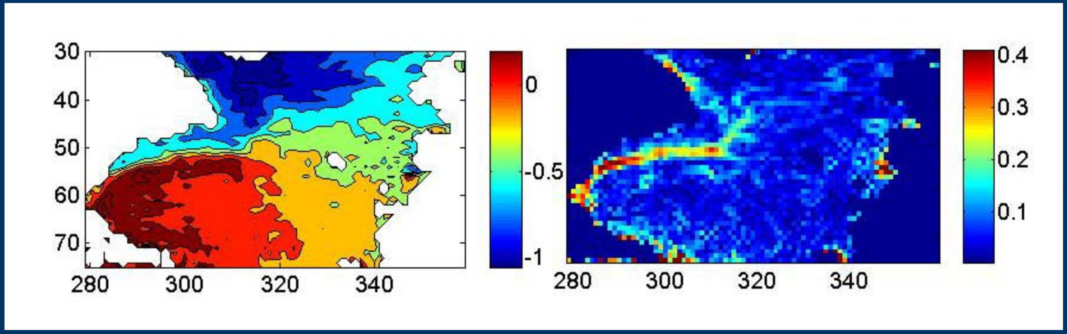
gravity as a global reference



low



medium



high

ocean topography and velocity field in the North Atlantic

conservation of linear momentum

$$\dot{u} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + 2\Omega v\sin\varphi + F_x$$

$$\dot{v} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - 2\Omega u\sin\varphi + F_y$$

$$\dot{w} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + 2\Omega u\cos\varphi - g + F_z$$

expressed in a local (spherical) coordinate system {east, north, up}
and rotating with the earth, p pressure, Ω earth angular velocity,
G gravity, F forces such as wind stress or tides

gravity as a global reference

$$\dot{u} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + 2\Omega v \sin\varphi + F_x$$

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$$\dot{w} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + 2\Omega u \cos\varphi - g + F_z$$

geostrophic balance:

pressure gradient = - Coriolis acceleration

and

hydrostatic (pressure) equation

gravity as a global reference

Scaling the Equations: The Geostrophic Approximation

We wish to simplify the equations of motion to obtain solutions that describe the deep-sea conditions well away from coasts and below the Ekman boundary layer at the surface. To begin, let's examine the typical size of each term in the equations in the expectation that some will be so small that they can be dropped without changing the dominant characteristics of the solutions. For interior, deep-sea conditions, typical values for distance L, horizontal velocity U, depth H, Coriolis parameter f, gravity g, and density ρ are:

$$L \approx 10^6 \text{ m} \qquad H_1 \approx 10^3 \text{ m} \qquad f \approx 10^{-4} \text{ s}^{-1} \qquad \rho \approx 10^3 \text{ kg/m}^3$$
$$U \approx 10^{-1} \text{ m/s} \qquad H_2 \approx 1 \text{ m} \qquad \rho \approx 10^3 \text{ kg/m}^3 \qquad g \approx 10 \text{ m/s}^2$$

where H_1 and H_2 are typical depths for pressure in the vertical and horizontal.

From these variables we can calculate typical values for vertical velocity W, pressure P, and time T:

$$\frac{\partial W}{\partial z} = -\left(\frac{\partial U}{\partial x} + \frac{\partial v}{\partial y}\right)$$
$$\frac{W}{H_1} = \frac{U}{L}; \quad W = \frac{U H_1}{L} = \frac{10^{-1}\,10^3}{10^6} \text{ m/s} = 10^{-4}\text{m/s}$$
$$P = \rho g H_1 = 10^3\,10^1\,10^3 = 10^7 \text{ Pa}; \quad \partial p/\partial x = \rho g H_2/L = 10^{-2}\text{Pa/m}$$
$$T = L/U = 10^7 \text{ s}$$

The momentum equation for vertical velocity is therefore:

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + 2\Omega\, u\cos\varphi - g$$
$$\frac{W}{T} + \frac{UW}{L} + \frac{UW}{L} + \frac{W^2}{H} = \frac{P}{\rho H_1} + fU - g$$
$$10^{-11} + 10^{-11} + 10^{-11} + 10^{-11} = 10 \quad + 10^{-5} - 10$$

and the only important balance in the vertical is hydrostatic:

$$\frac{\partial p}{\partial z} = -\rho g \quad \text{Correct to} \quad 1:10^6.$$

The momentum equation for horizontal velocity in the x direction is:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv$$
$$10^{-8} + 10^{-8} + 10^{-8} + 10^{-8} = 10^{-5} + 10^{-5}$$

Thus the Coriolis force balances the pressure gradient within one part per thousand. This is called the *geostrophic balance*, and the *geostrophic equations* are:

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = fv; \quad \frac{1}{\rho}\frac{\partial p}{\partial y} = -fu; \quad \frac{1}{\rho}\frac{\partial p}{\partial z} = -g$$

This balance applies to oceanic flows with horizontal dimensions larger than roughly 50 km and times greater than a few days.

scaling of the momentum equations

leads to the

geostrophic balance

Robert H Stewart:
Introduction to Physical Oceanography, 2008

theory

gravity as a global reference

$$\dot{u} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + 2\Omega v \sin\varphi + F_x$$

$$\dot{v} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - 2\Omega u \sin\varphi + F_y$$

$$\dot{w} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + 2\Omega u \cos\varphi - g + F_z$$

geostrophic balance:

pressure gradient = - Coriolis acceleration

and

hydrostatic (pressure) equation

$$\partial p = -g\rho\partial z \quad \succ$$

$$g\frac{\partial z}{\partial x} = -2\Omega\sin\varphi\, v \quad or \quad \frac{\partial H}{\partial x} = -\frac{f}{g}v$$

$$g\frac{\partial z}{\partial y} = 2\Omega\sin\varphi\, u \quad or \quad \frac{\partial H}{\partial y} = \frac{f}{g}u$$

theory

gravity as a global reference

$$\partial p = -g\rho\,\partial z \quad \succ$$

$$g\frac{\partial z}{\partial x} = -2\Omega\sin\varphi\, v \qquad or \qquad \frac{\partial H}{\partial x} = -\frac{f}{g}v$$

$$g\frac{\partial z}{\partial y} = 2\Omega\sin\varphi\, u \qquad or \qquad \frac{\partial H}{\partial y} = \frac{f}{g}u$$

estabishes the relationship between sea surface slope

{δH/δx , δH/δy}

and surface ocean circulation (velocity);

the motion is perpendicular to the slope i.e. parallel to the contour lines of DOT;

the slope is proportional to the velocity

gravity as a global reference

A connection between themes Two and Three:

from surface circulation to ocean velocity at depth

by measuring temperature and salinity profiles

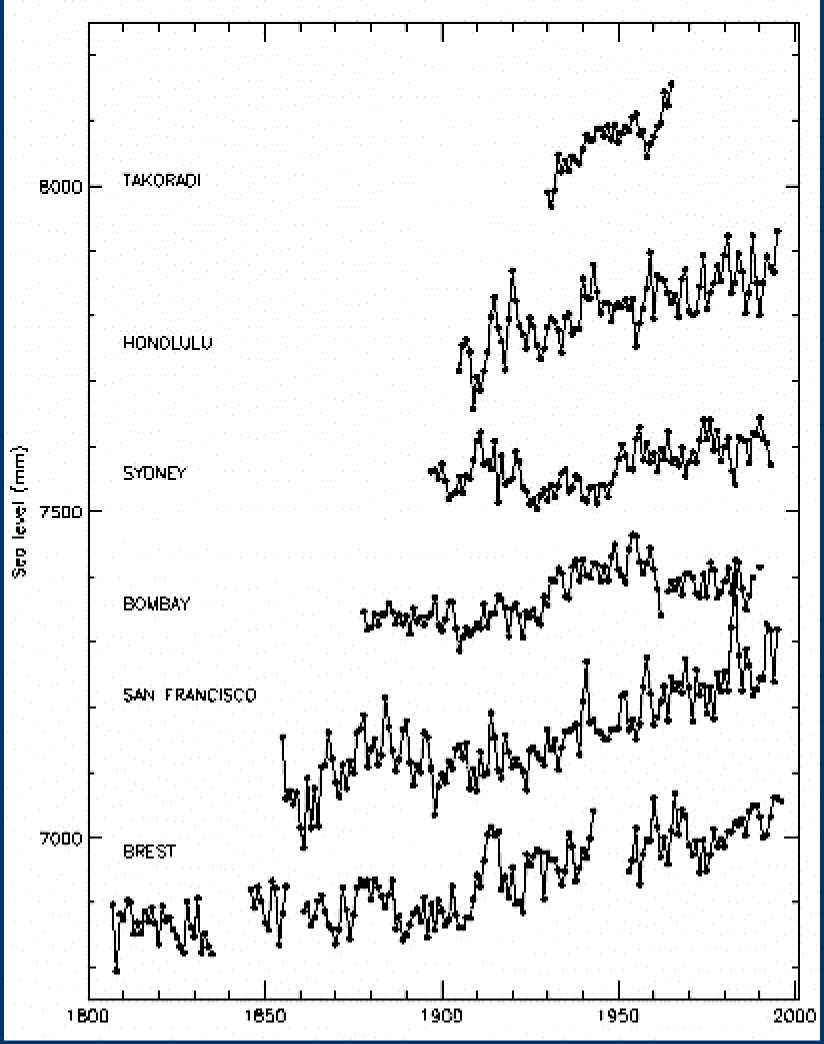
(or vertical changes of pressure)

$$u = -\frac{1}{f\rho}\frac{\partial}{\partial y}\int_{depth}^{0} g(\varphi, z)\rho(z)\,dz - \frac{g}{f}\frac{\partial H}{\partial y}$$

$$v = -\frac{1}{f\rho}\frac{\partial}{\partial x}\int_{depth}^{0} g(\varphi, z)\rho(z)\,dz + \frac{g}{f}\frac{\partial H}{\partial x}$$

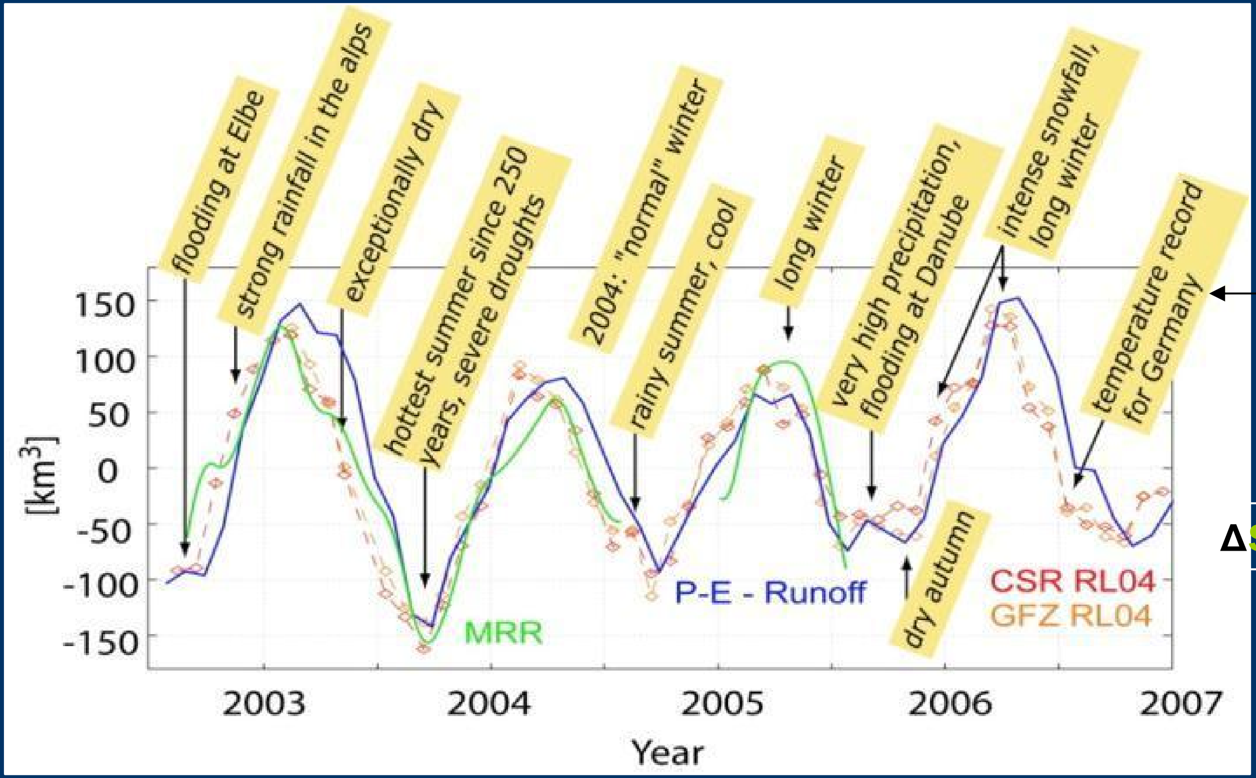
gravity as a global reference

relative sea level at six tide gauges



from individual
records
to a global process

Theme 3: temporal variations of gravity



ΔS(GRACE)

Seitz et al., EPSL,2008

(sub-)seasonal water storage variability in Central Europe (from GRACE)

temporal gravity and mass exchange

principle:

- measurement of gravity and/or geoid change
  by satellite gravimetry
- measurement of changes of surface geometry

on land:	GNSS (GPS, GALILEO…) and InSAR
over ice:	ice altimetry (CRYOSAT-2, ICESAT) and InSAR
on ocean:	radar altimetry

temporal gravity and mass exchange

from pressure to surface layer to equivalent water height

$$\left.\begin{matrix} \rho_{atm}\Delta r_{atm} \\ \rho_{ocean}\Delta r_{ocean} \\ \rho_{ice}\Delta r_{ice} \\ \rho_{rock}\Delta r_{rock} \end{matrix}\right\} = \Delta\sigma = \rho_{water}\Delta h_{EWH}$$

temporal gravity and mass exchange

$$\Delta\sigma(\varphi,\lambda) = \int_{\text{thinlayer}} \Delta\rho(\varphi,\lambda,r)\,dr$$

$$\left.\begin{matrix}\delta\bar{C}_{nm}\\ \delta\bar{S}_{nm}\end{matrix}\right\} = \frac{1}{R\rho_{water}}\frac{1}{4\pi}\int_0^{2\pi}\int_{-\pi/2}^{\pi/2}\Delta\sigma(\varphi,\lambda)\bar{P}_{nm}(\varphi)\left\{\begin{matrix}\cos m\lambda\\ \sin m\lambda\end{matrix}\right\}\cos\varphi\, d\varphi\, d\lambda$$

satellite gravimetry

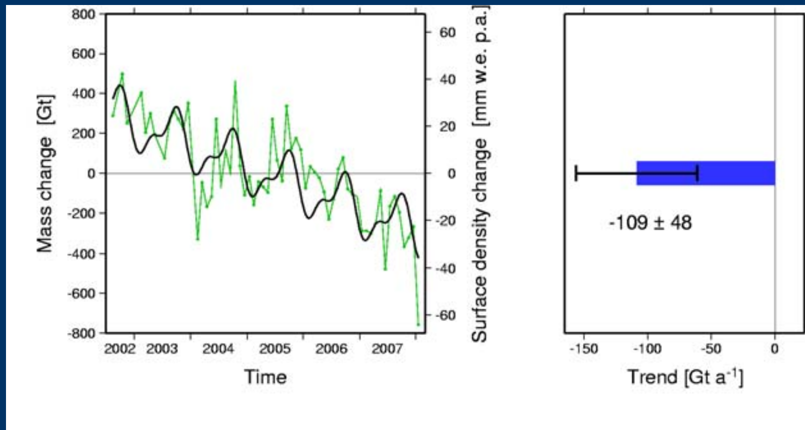
measures temporal changes of the

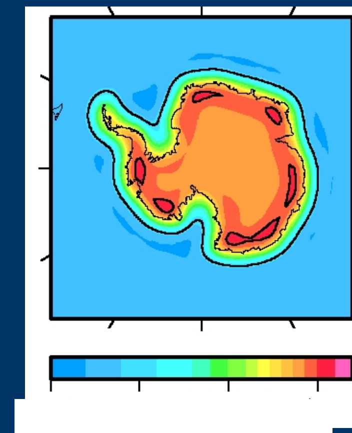
spherical harmonic coefficients δC_{nm} and δS_{nm}

$$\Delta\sigma(\varphi,\lambda) = R\rho_{water}\sum_{n=0}^{\infty}\sum_{m=0}^{n}\bar{P}_{nm}(\varphi)\left(\delta\bar{C}_{nm}\cos m\lambda + \delta\bar{S}_{nm}\sin m\lambda\right)$$

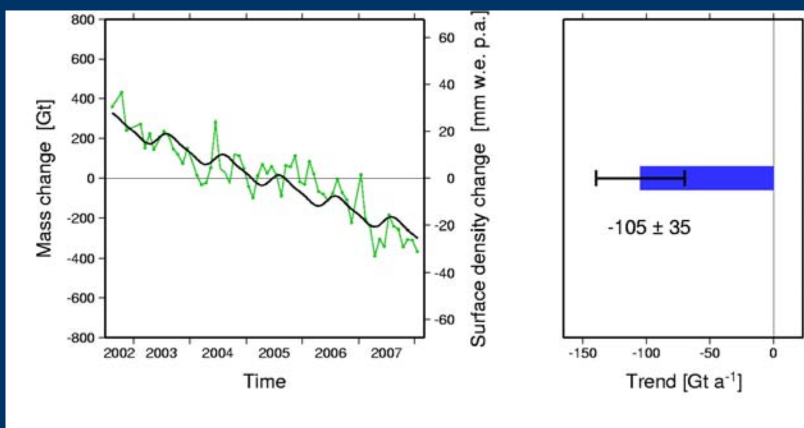
Wahr & Molenaar, JGR (red), 1998

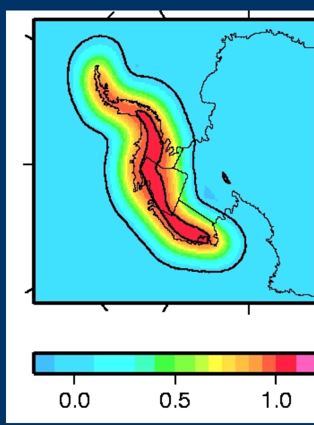
temporal gravity and mass exchange





weight function





weight function

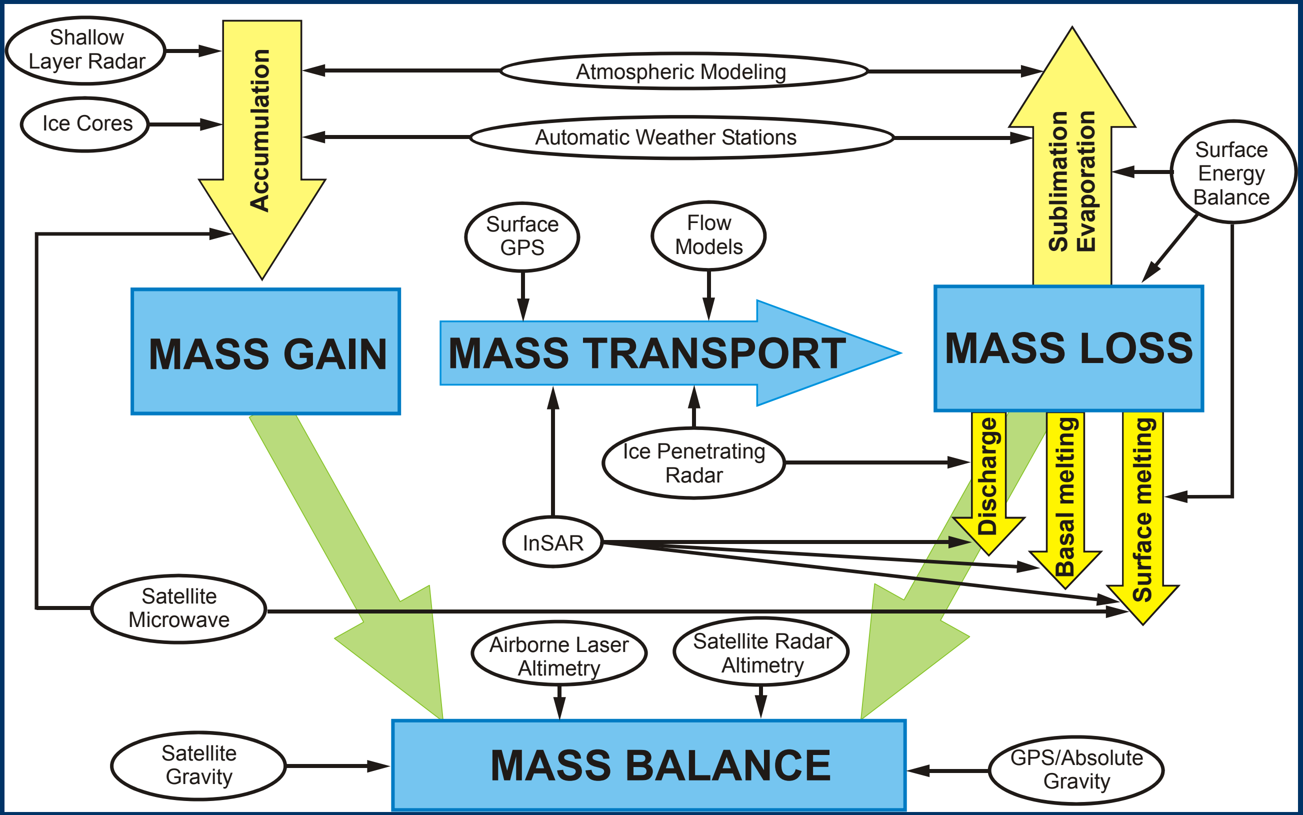
example 1: mass loss in Antarctica

Horwath and Dietrich, 2009, Geophys. J. Int., 2009

temporal gravity and mass exchange

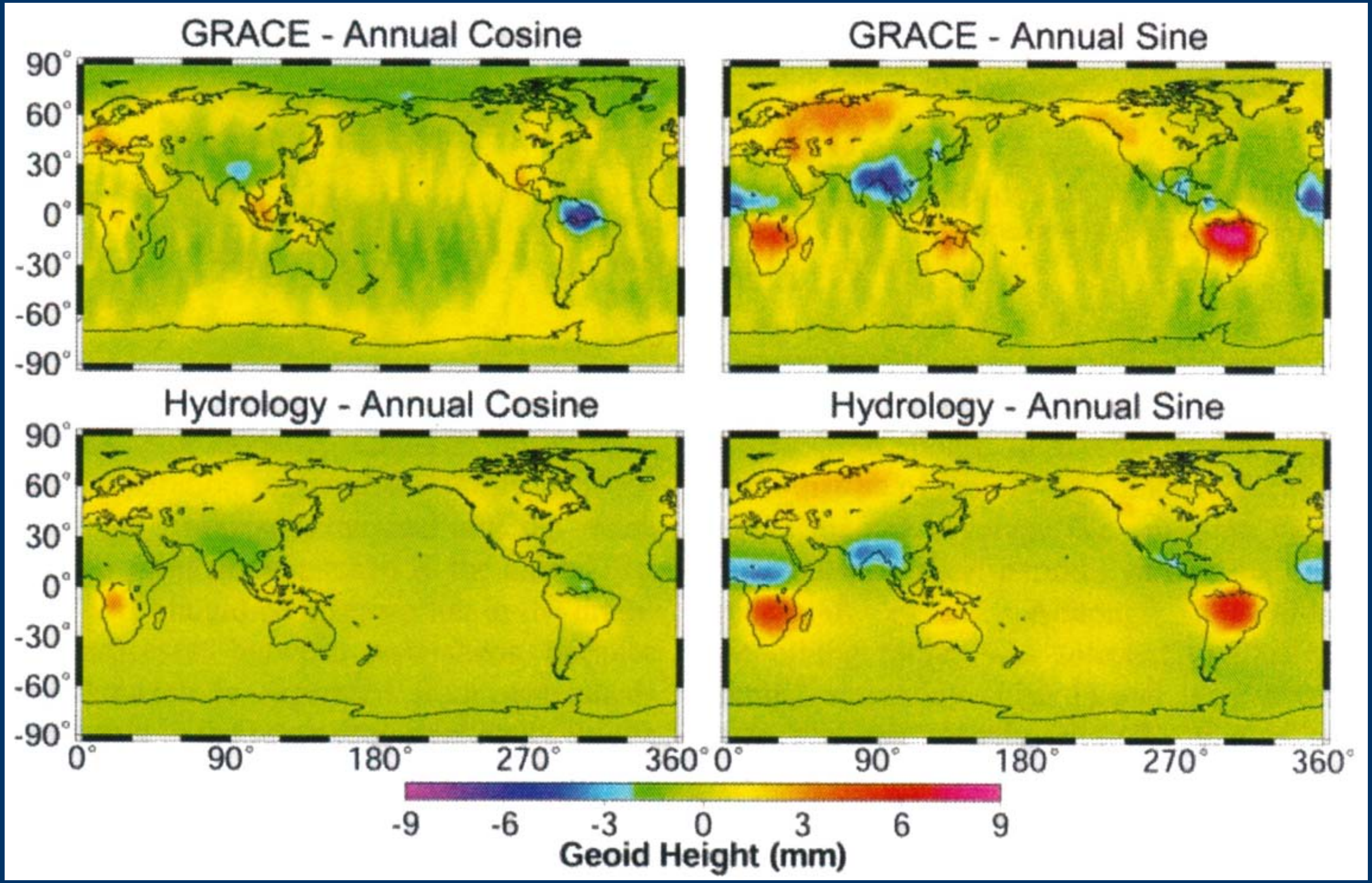
thematic (geodetic) observing system:

ice mass balance



Thomas, 2001, EOS

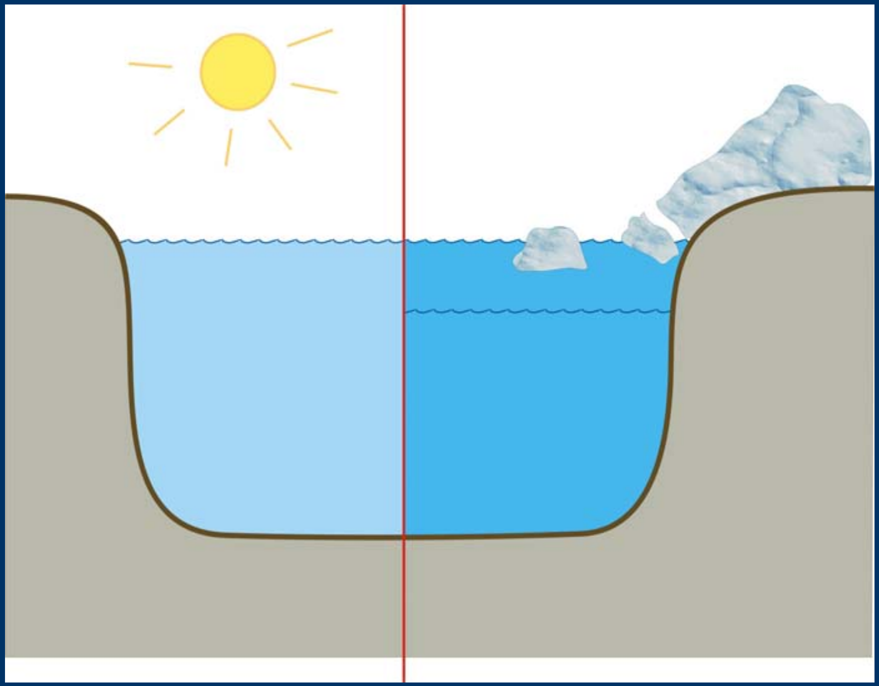
temporal gravity and mass exchange



Tapley et al., Science, 2004

example 2: seasonal variation of continental water storage

temporal gravity and mass exchange



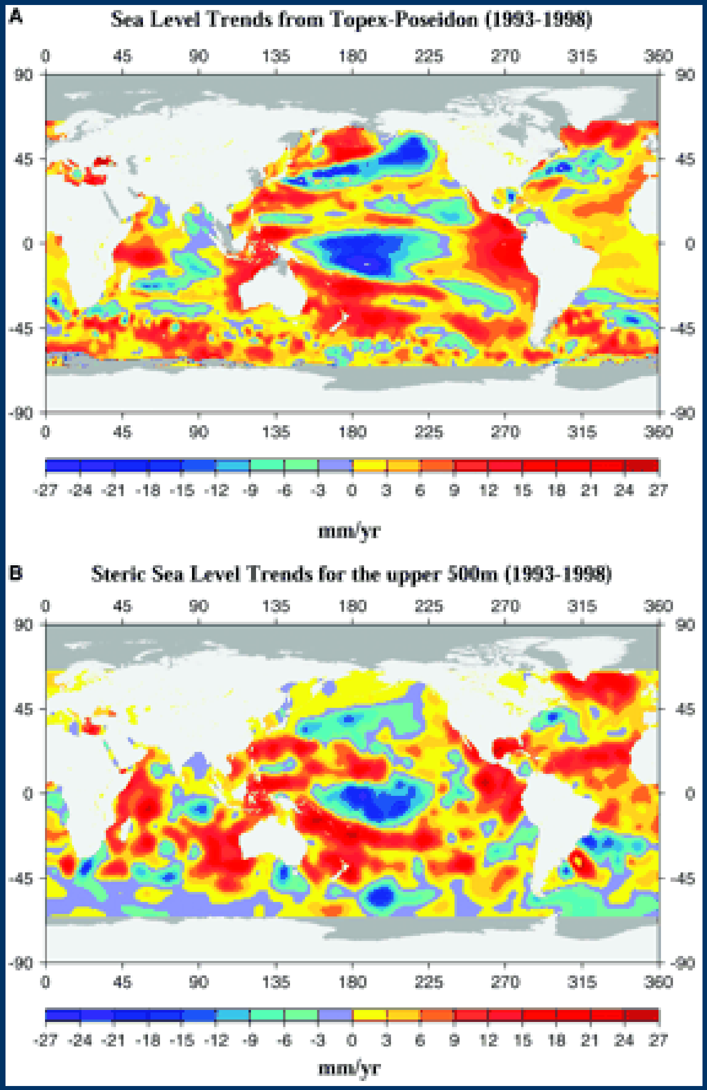
example 3: the causes of sea level change

separation of mass gain due to ice melting from thermal expansion (steric effect):

mass gain: strong gravity signal

thermal expansion: negligible gravity signal

temporal gravity and mass exchange



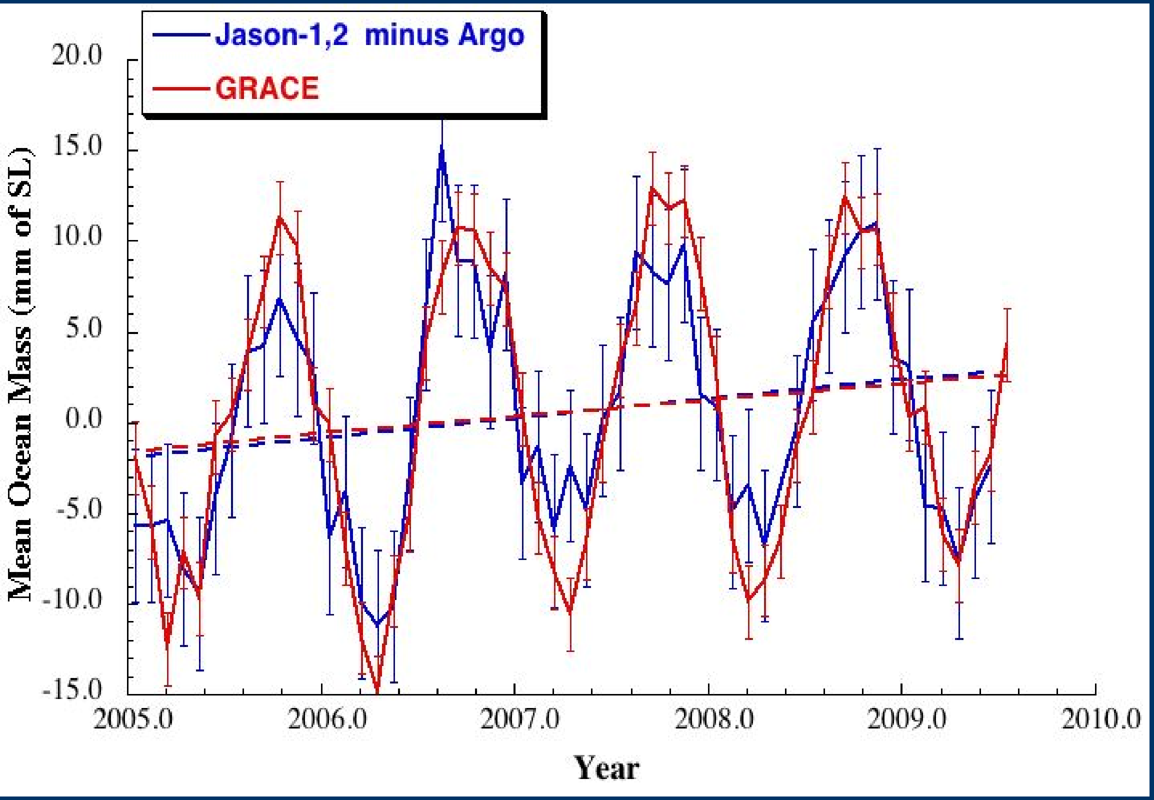
sea level change
from satellite altimetry
(T/P altimetry, 1993-1998)

steric sea level change
thermal expansion
(temperature, 500m, 1993-1998)

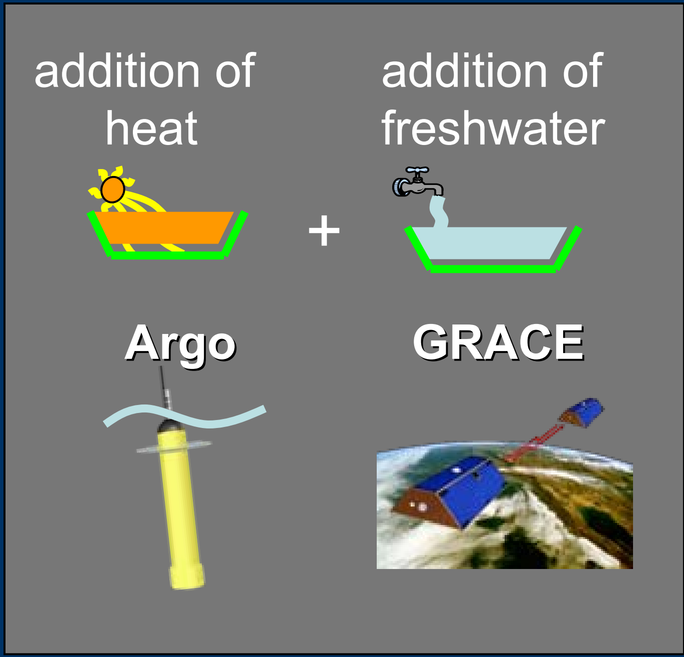
Cabanes, Cazenave & LeProvost, 2001

temporal gravity and mass exchange

sea level budget 2004-2009



Chambers, 2009



GRACE trend (2003-2009.5) = 1.3 ± 0.8 mm/yr

temporal gravity and mass exchange

A connection between themes Two and Three:
from surface circulation to ocean velocity at depth
by measuring temperature and salinity profiles
(or vertical changes of pressure)

$$u = -\frac{1}{f\rho}\frac{\partial}{\partial y}\int_{depth}^{0} g(\varphi,z)\rho(z)\,dz - \frac{g}{f}\frac{\partial H}{\partial y}$$

$$v = -\frac{1}{f\rho}\frac{\partial}{\partial x}\int_{depth}^{0} g(\varphi,z)\rho(z)\,dz + \frac{g}{f}\frac{\partial H}{\partial x}$$

temporal gravity and mass exchange

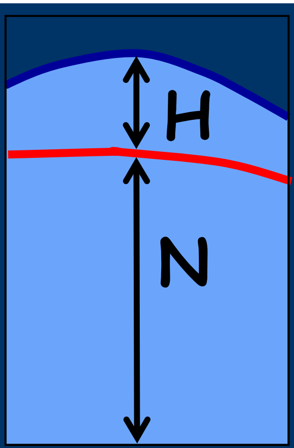
gravity und oceanography:

geoid plus altimety =

dynamic ocean topography =

surface ocean circulation

⇨

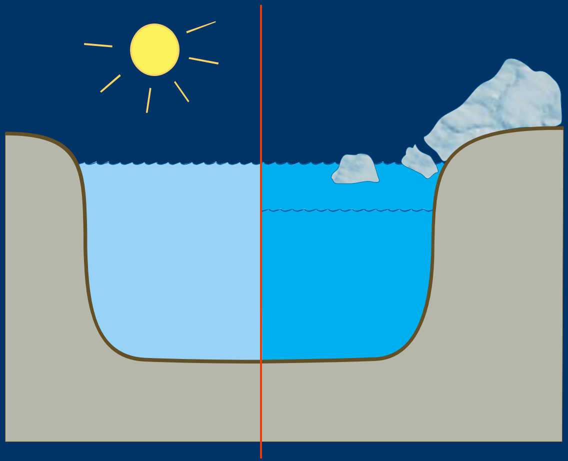


temporal gravity variation

and sea level change:

thermal expansion versus mass increase

⇨

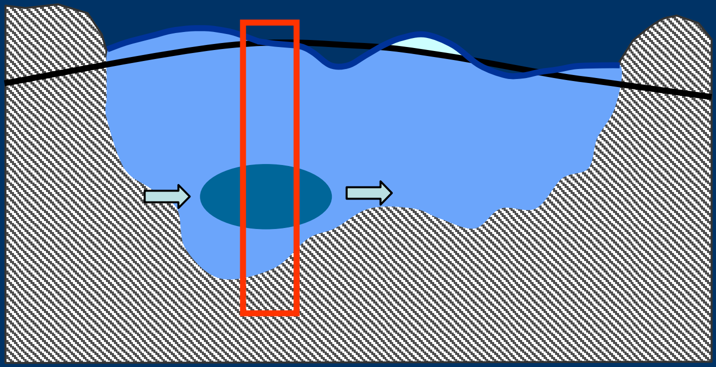


temporal gravity variation =

bottom pressure variation =

deep ocean circulation

⇨



Summary of lecture Two

1. There are three uses of gravity for earth system studies:
   - gravity anomalies and their relationship to interior density and geodynamics,
   - the geoid as a global reference level surface for ocean circulation studies and determination of mass and heat transport, and
   - temporal changes of gravity and geoid as estimates of (climatic) mass exchange processes.
2. Inference of density from gravity or geoid anomalies poses an inverse problem The answer is joint inversion with seismic tomography, and other geodynamic data (topography, crustal motion, earth rotation, laboratory experiments…).
3. Mean ocean surface from multi-year satellite altimetry and the geoid yield dynamic ocean topography. Geostrophic equations permit the derivation of global surface circulation (changes).
4. Measurement of temporal geoid/ gravity changes is a new type of global parameter set used in climate change studies (global water cycle, global ice mass balance, causes of sea level change, glacial isostatic adjustment).