Introduction to Data Assimilation

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What is data assimilation?

Data assimilation is the technique whereby observational data are combined with output from a numerical model to produce an optimal estimate of the evolving state of the system.

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Why We Need Data Assimilation

- range of observations
- range of techniques
- different errors
- data gaps
- quantities not measured
- quantities linked

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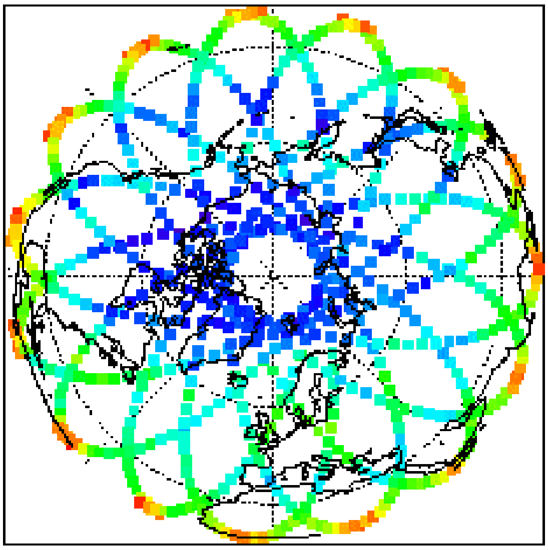

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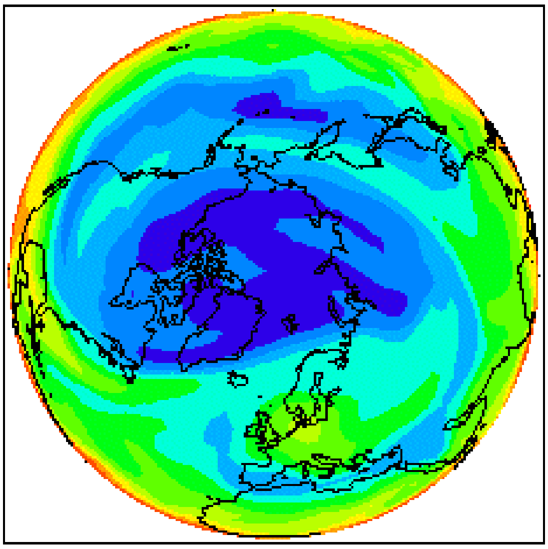

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Some Uses of Data Assimilation

- Operational weather and ocean forecasting
- Seasonal weather forecasting
- Land-surface process
- Global climate datasets
- Planning satellite measurements
- Evaluation of models and observations

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What We Want To Know

$\mathbf{x}(t)$ atmos. state vector

$\mathbf{s}(t)$ surface fluxes

$\mathbf{c}$ model parameters

$$\mathbf{X}(t) = (\mathbf{x}(t), \mathbf{s}(t), \mathbf{c})$$

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What We Also Want To Know

Errors in models

Errors in observations

What observations to make

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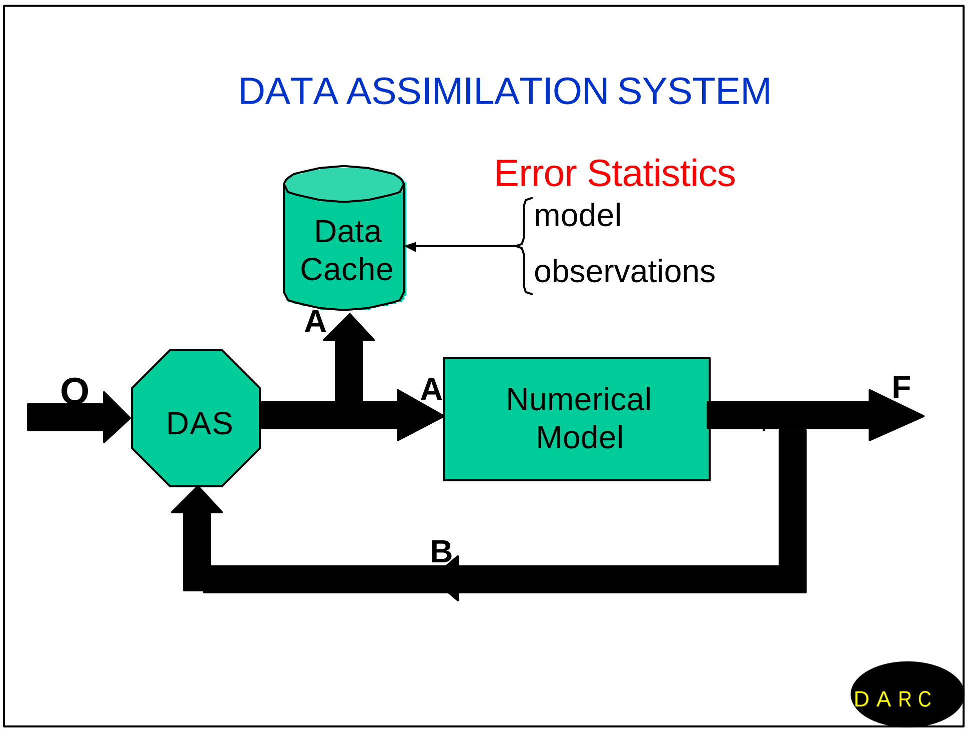
DATA ASSIMILATION SYSTEM
Error Statistics
model
observations
Data Cache
A
O
DAS
A
Numerical Model
F
B
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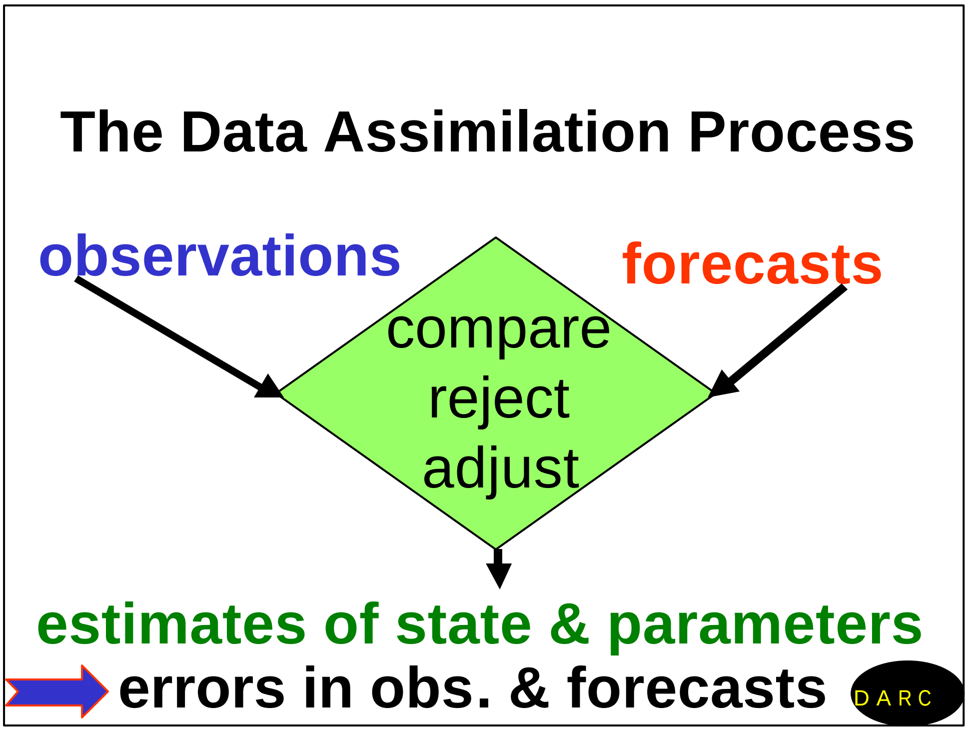
The Data Assimilation Process
observations
forecasts
compare
reject
adjust
estimates of state & parameters
errors in obs. & forecasts
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Retrievals and Assimilation

- Problems with retrievals
  - *a priori* state and poorly known errors
- Problems with assimilation of radiance
  - systematic errors and cloud clearing
  - expensive (multi-channels)
- Need effective interface
  - Information content with error analysis

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Statistical Approach to Data Assimilation

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Minimum Variance

Combination of Data

Unbiased, Uncorrelated Errors

$$\tilde{x} = \alpha x_1 + \beta x_2 \qquad \alpha + \beta = 1$$

$$\mathrm{Var}(x_1) = \sigma_1^2 \qquad \mathrm{Var}(x_2) = \sigma_2^2$$

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Minimum Variance Combination of Data Unbiased, Uncorrelated Errors

$$\mathrm{Var}(\tilde{x}) = \alpha^2 \sigma_1^2 + (1-\alpha)^2 \sigma_2^2$$

$$\frac{\partial}{\partial \alpha} \mathrm{Var}(\tilde{x}) = 2\alpha\sigma_1^2 - 2(1-\alpha)\sigma_2^2 = 0$$

$$\Rightarrow \alpha, \beta$$

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Minimum Variance Combination of Data Unbiased, Uncorrelated Errors

$$\tilde{x} = \left(\frac{x_1}{\sigma_1^2} + \frac{x_2}{\sigma_2^2}\right)\left(\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}\right)^{-1}$$

$$\mathrm{var}(\tilde{x}) \leq \min(\sigma_1^2, \sigma_2^2)$$

Best Linear Unbiased Estimate

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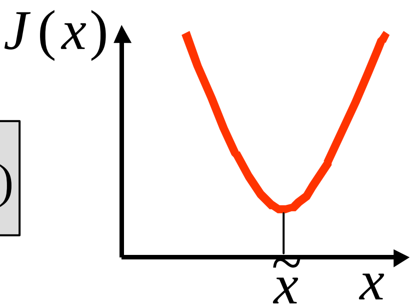
Variational Method

$$J(x) = \frac{(x - x_1)^2}{\sigma_1^2} + \frac{(x - x_2)^2}{\sigma_2^2}$$

$\tilde{x}$ at $\min J(x)$



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Maximum Likelihood Estimate

- Obtain or assume probability distributions for the errors
- The best estimate of the state is chosen to have the greatest probability, or maximum likelihood
- If errors normally distributed,unbiased and uncorrelated, then states estimated by minimum variance and maximum likelihood are the same

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Multivariate Case

state vector $\mathbf{x}(t) = \begin{pmatrix} x_1 \\ x_2 \\ \bullet \\ \bullet \\ x_n \end{pmatrix}$

observation vector $\begin{pmatrix} y_1 \\ y_2 \\ \bullet \\ y_m \end{pmatrix}$

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Variance becomes Covariance Matrix

- **Errors in x_i are often correlated**
  - **spatial structure in flow**
  - **dynamical or chemical relationships**
- **Variance for scalar case becomes Covariance Matrix for vector case** COV
- **Diagonal elements are the variances of x_i**
- **Off-diagonal elements are covariances between x_i and x_j**
- **Observation of x_i affects estimate of x_j**

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Estimating Covariance Matrix for Observations, O

- O usually quite simple:
  - diagonal or
  - for nadir-sounding satellites, non-zero values between points in vertical only
- Calibration against independent measurements

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Estimating Covariance Matrix for Model, B

- Use simple analytical functions constructed experimentally, e.g.

$$B_{ij} = \sigma_i \sigma_j \exp(-d_{ij})$$

where d_{ij} is the horizontal distance between $\mathbf{x}_i$ and $\mathbf{x}_j$

- Run ensemble of forecasts from slightly different conditions and analyse error growth.

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Methods of Data Assimilation

- Optimal interpolation (or approx. to it)
- 3D variational method (3DVar)
- 4D variational method (4DVar)
- Kalman filter (with approximations)

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Optimal Interpolation

observation

observation operator

$$\mathbf{x}_a = \mathbf{x}_b + \mathbf{K}(\mathbf{y} - H(\mathbf{x}_b))$$

"analysis"

"background" (forecast)

- linearity $H \rightarrow \mathrm{H}$
- matrix inverse
- limited area

$$\mathbf{K} = \mathbf{B}\mathbf{H}^T(\mathbf{H}\mathbf{B}\mathbf{H}^T + \mathbf{O})^{-1}$$

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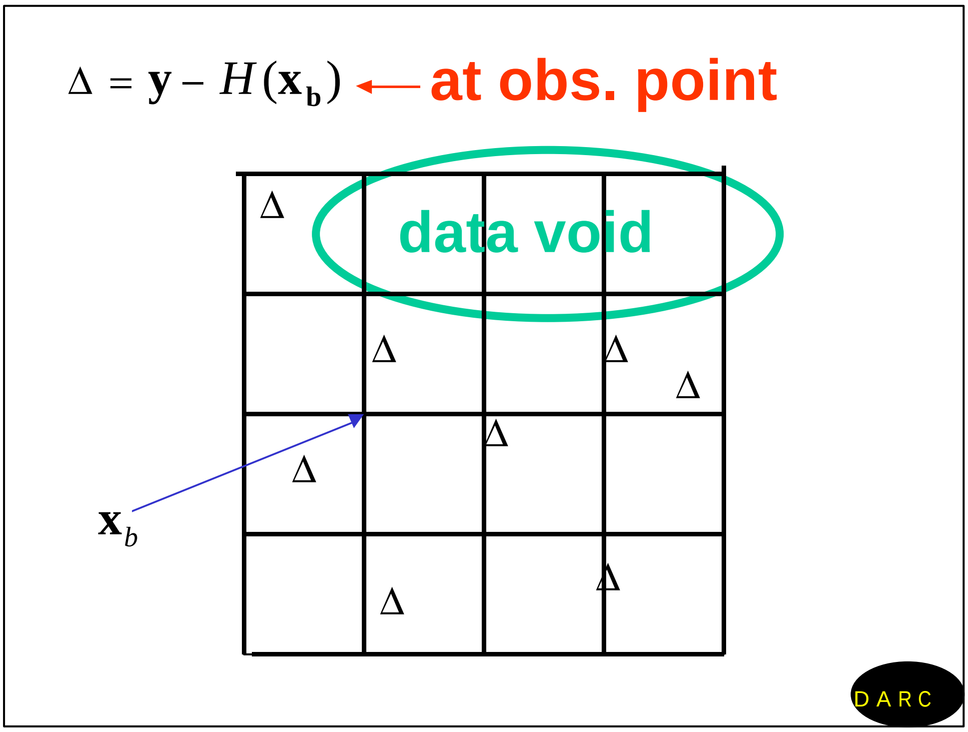
$\Delta = \mathbf{y} - H(\mathbf{x}_b)$ ← at obs. point
data void
Δ
$\mathbf{x}_b$
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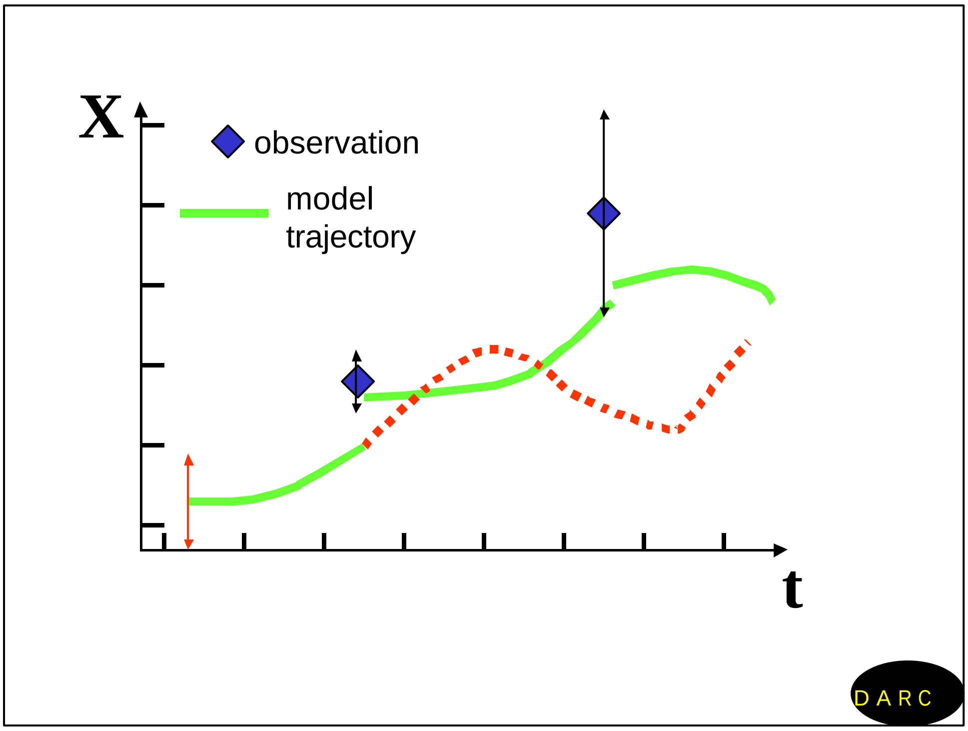
X
observation
model
trajectory
t
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Data Assimilation: an analogy

Driving with your eyes closed:

open eyes every 10 seconds and correct trajectory

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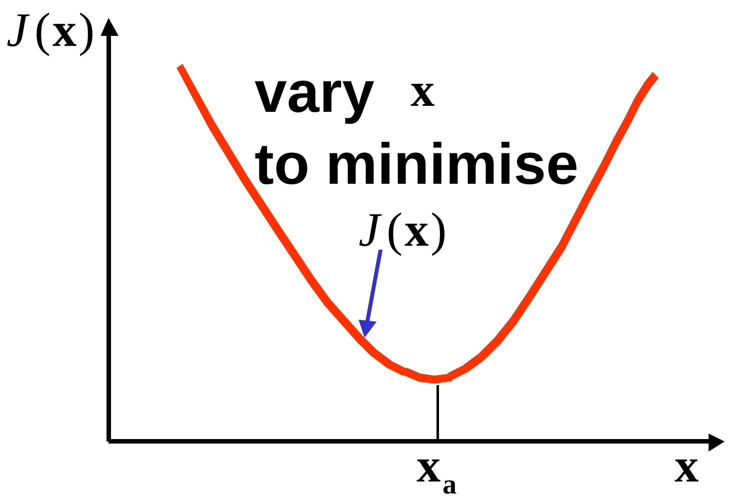
Variational Data Assimilation



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Variational Data Assimilation

$$J(\mathbf{x}) =$$
$$(\mathbf{x}-\mathbf{x}_b)^T \mathbf{B}^{-1} (\mathbf{x}-\mathbf{x}_b) +$$
$$(\mathbf{y}-H(\mathbf{x}))^T \mathbf{O}^{-1} (\mathbf{y}-H(\mathbf{x}))$$

nonlinear operator

assimilate y directly

global analysis

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Choice of State Variables and Preconditioning

- **Free to choose which variables to use to define state vector, *x(t)***
- **We'd like to make B diagonal**
  - **may not know *covariances* very well**
  - **want to make the minimization of J more efficient by "preconditioning": transforming variables to make surfaces of constant J nearly spherical in state space**

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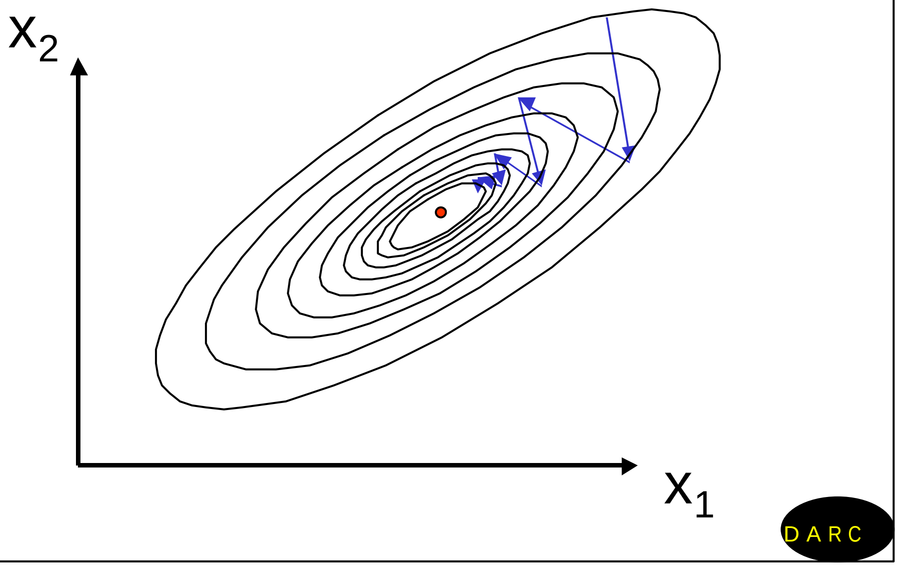
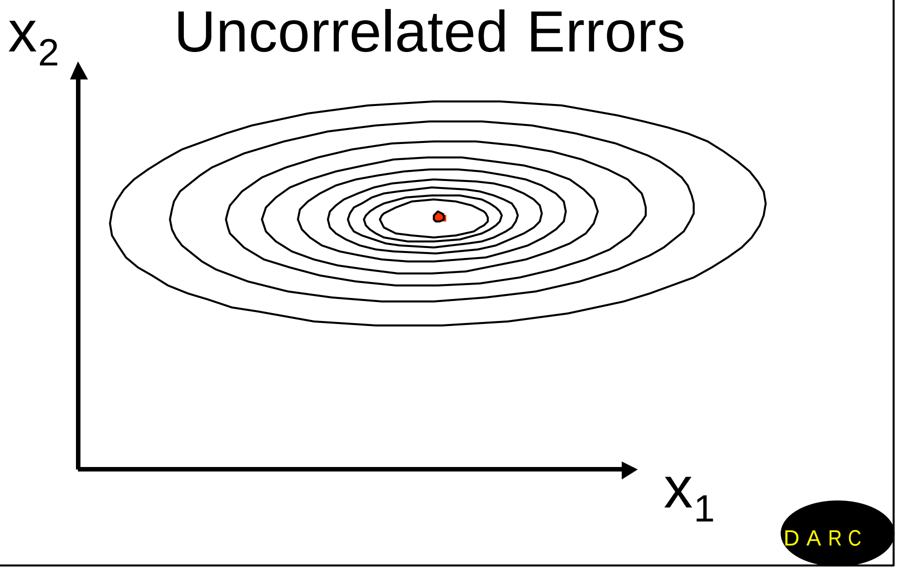
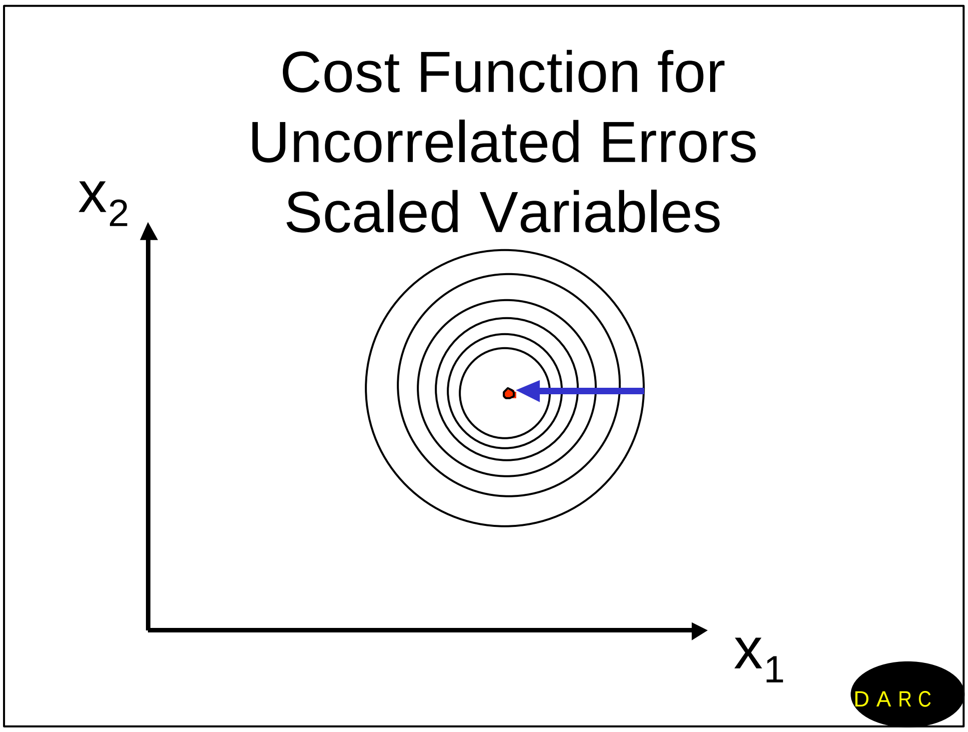
Cost Function for Correlated Errors



Cost Function for Uncorrelated Errors




Cost Function for
Uncorrelated Errors
Scaled Variables
X2
X1
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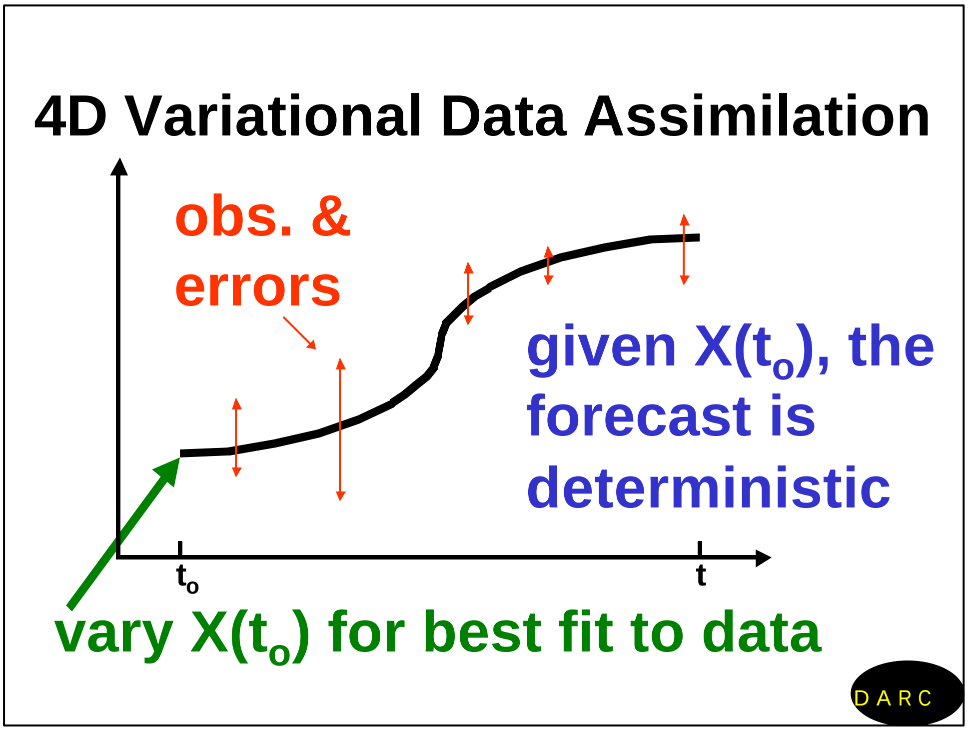
4D Variational Data Assimilation
obs. &
errors
given X(to), the
forecast is
deterministic
to
t
vary X(to) for best fit to data
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4D Variational Data Assimilation

- **Advantages**
  - consistent with the governing eqs.
  - implicit links between variables
- **Disadvantages**
  - very expensive
  - model is strong constraint

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Problem

model error covariances

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Kalman Filter
(*expensive*)

Use model equations to propagate B forward in time.

B ⟶ B(t)   KAL

Analysis step as in OI

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What are the benefits of data assimilation?

- Quality control
- Combination of data
- Errors in data and in model
- Filling in data poor regions
- Designing observational systems
- Maintaining consistency
- Estimating unobserved quantities

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Some Applications of Data Assimilation

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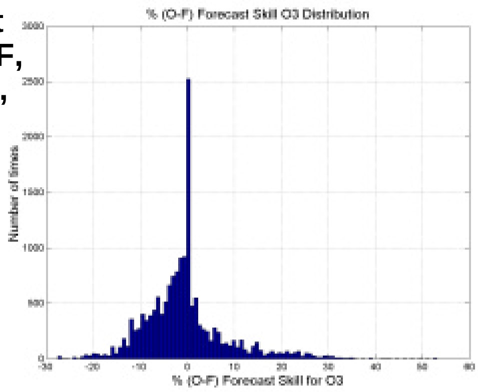
Skill Measures: Observation Increment, (O-F)

- The difference between the forecast from the first guess, F, and the observations, O, also known as observed-minus-background differences or the innovation vector.
- This is probably the best measure of forecast skill.



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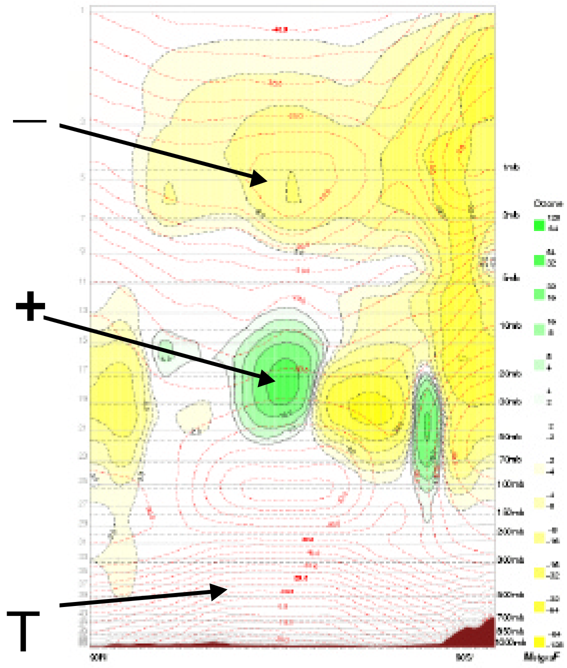

ECMWF

Ozone monthly-mean analysis increment.

Sept. 1986

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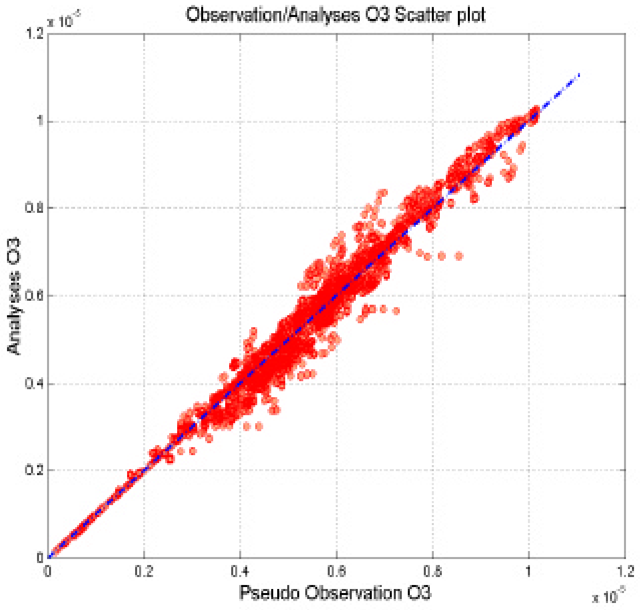

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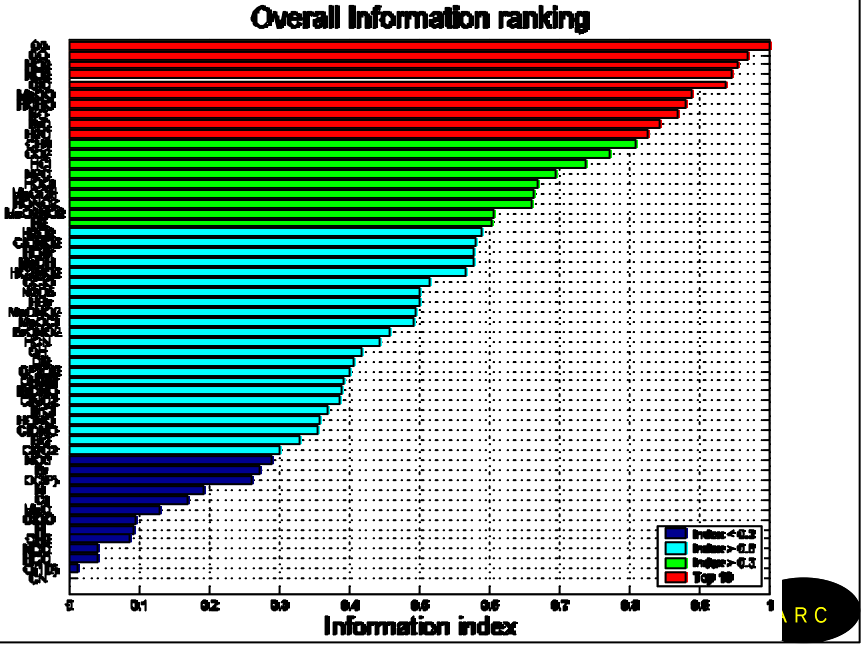
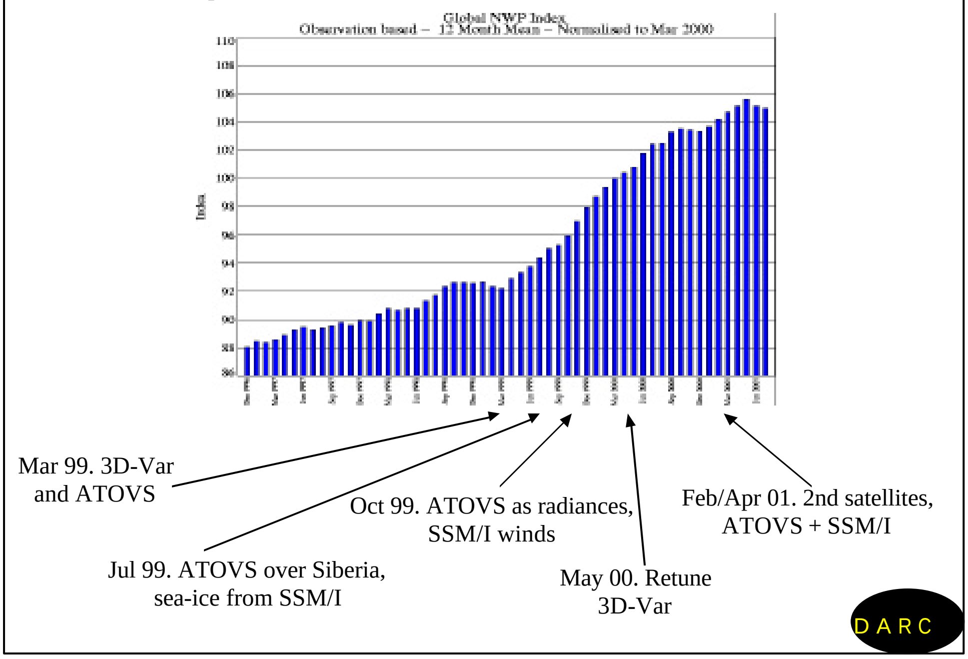
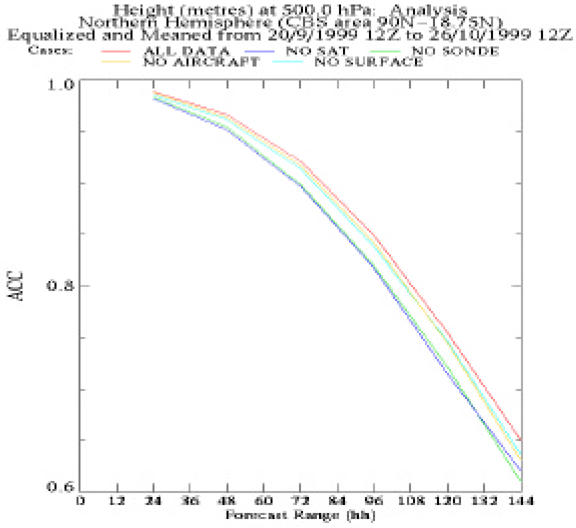
Best observations to make to characterize the chemical system



Impact on NWP at the Met Office



Impact of satellite data in NWP



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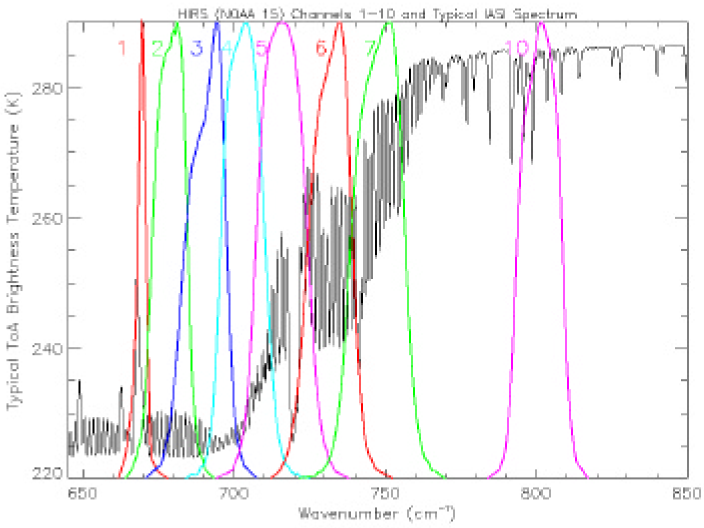
Future Advanced Infrared Sounders



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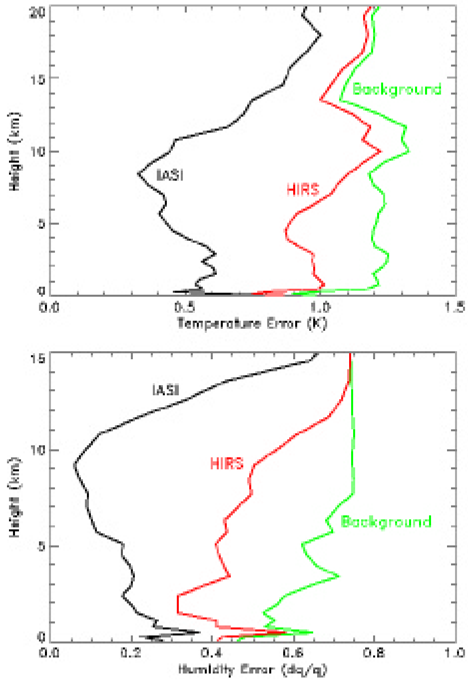

IASI vs HIRS

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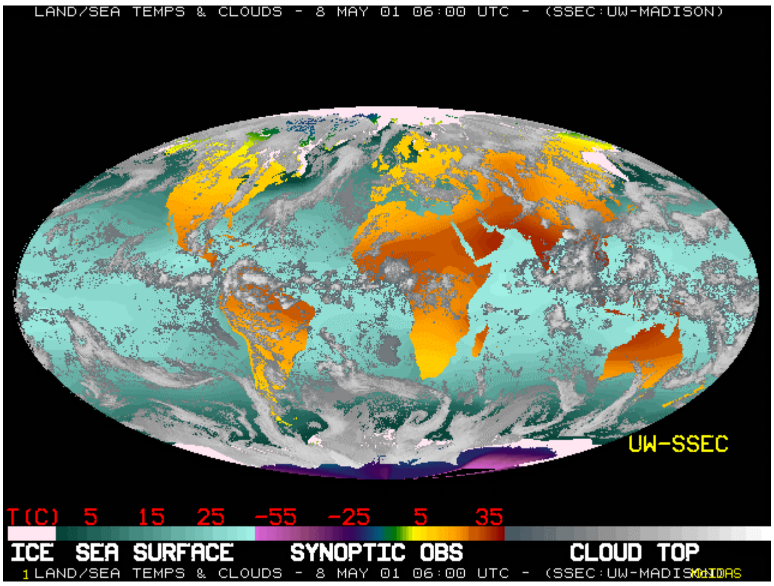

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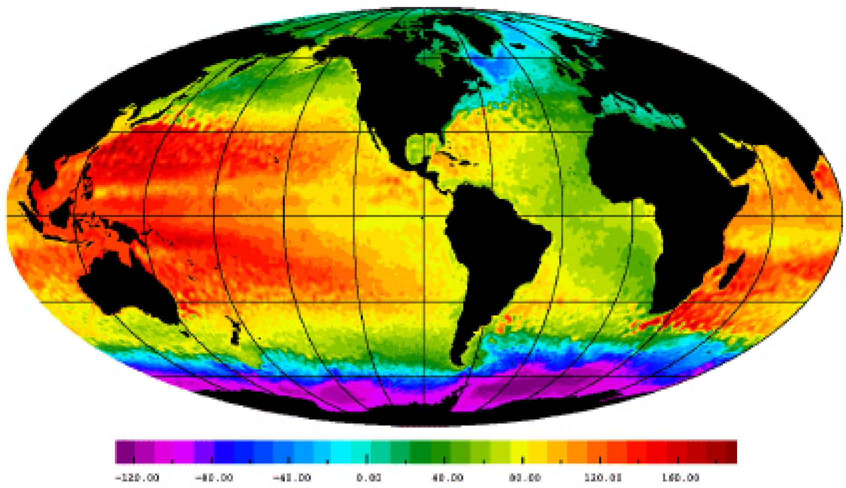

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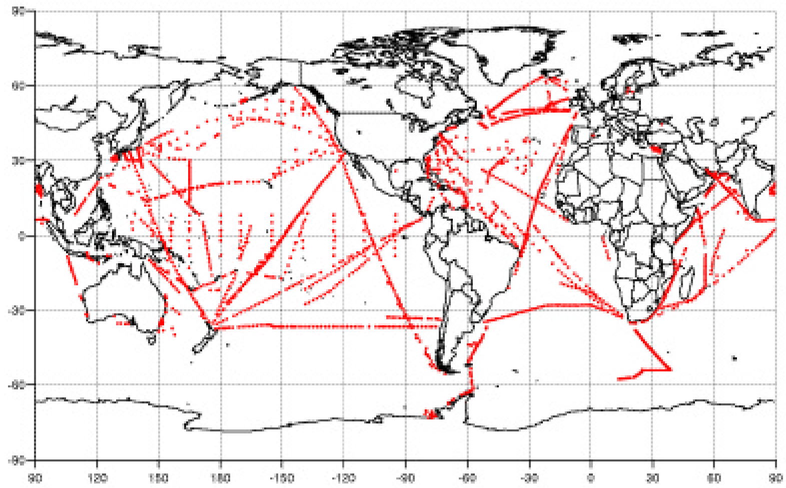
XBT data assimilated in March 1996.



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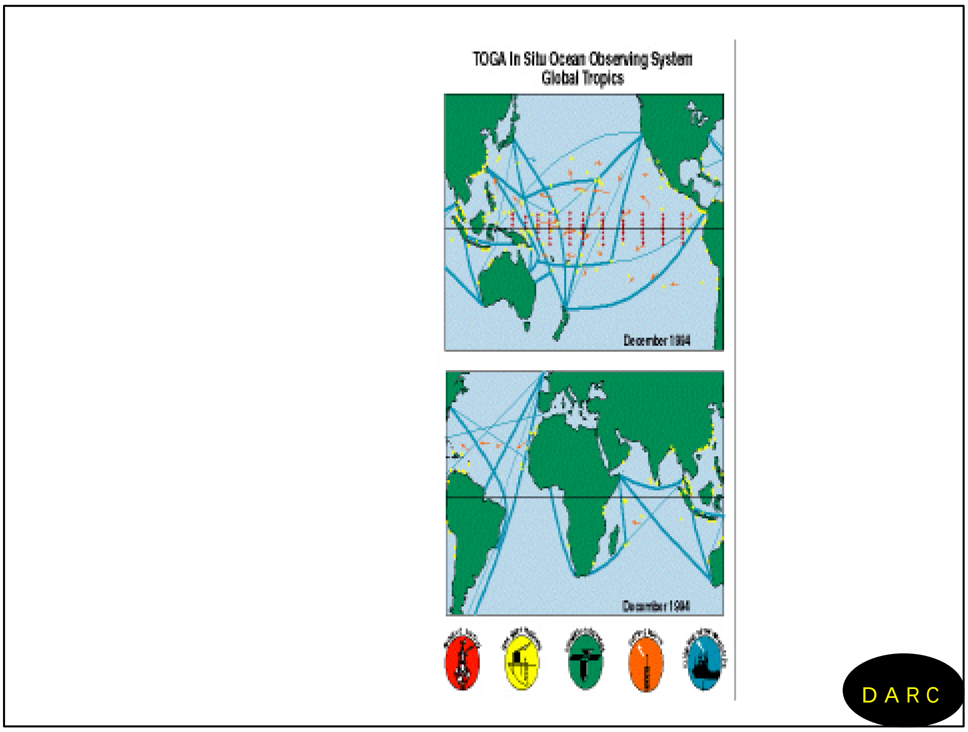
TOGA In Situ Ocean Observing System
Global Tropics
December 1994
December 1994
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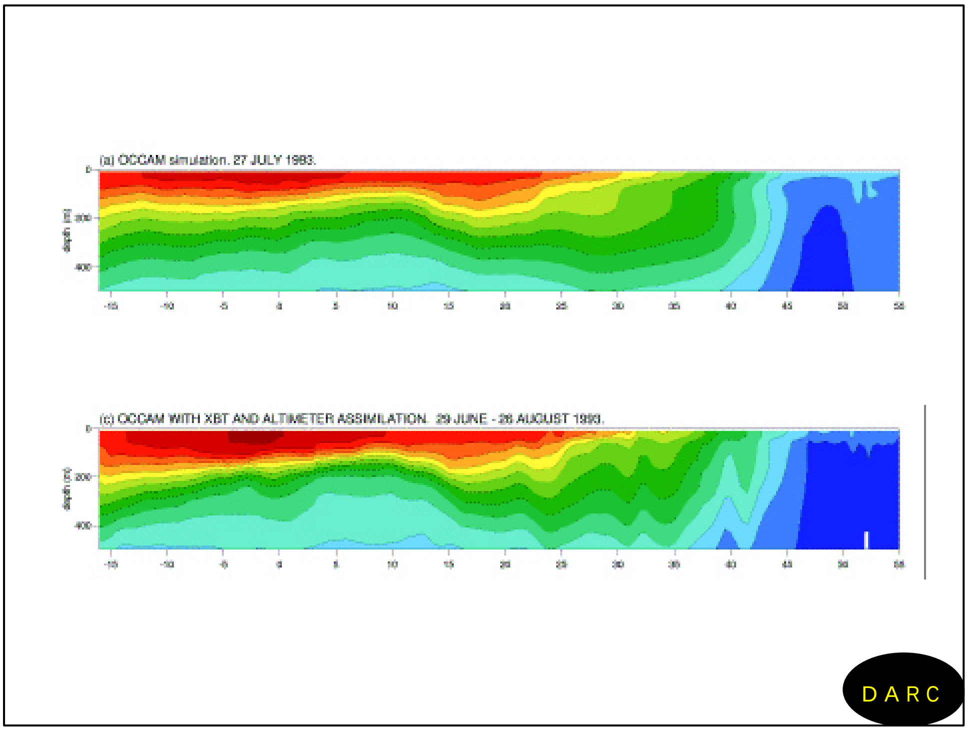
(a) OCCAM simulation. 27 JULY 1993.
(c) OCCAM WITH XBT AND ALTIMETER ASSIMILATION. 29 JUNE - 28 AUGUST 1993.
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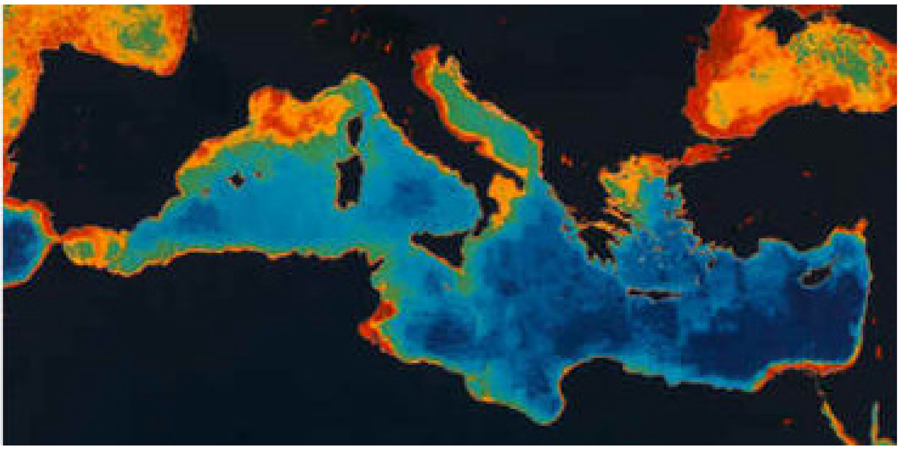
MERIS ocean colour



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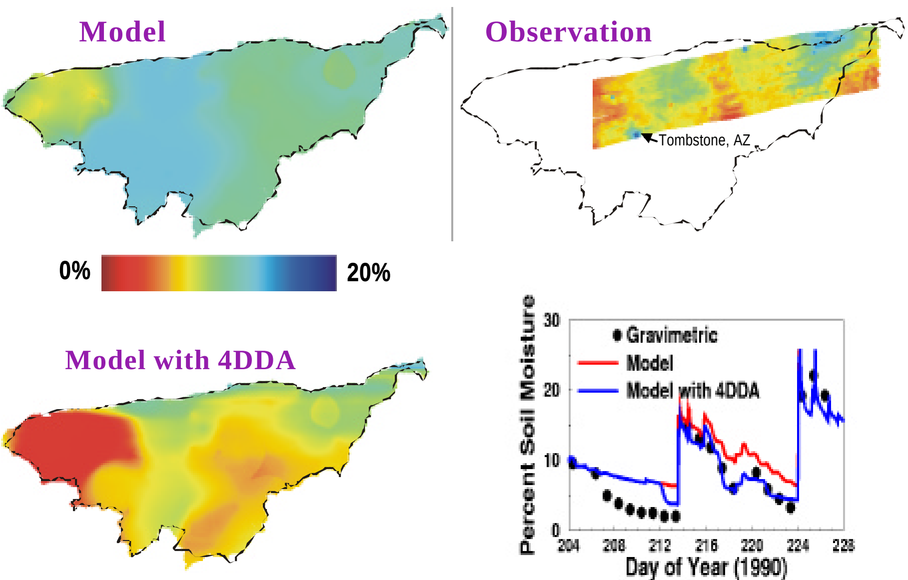
Regional Scale: Walnut Gulch (Monsoon 90)



Houser et al., 1998

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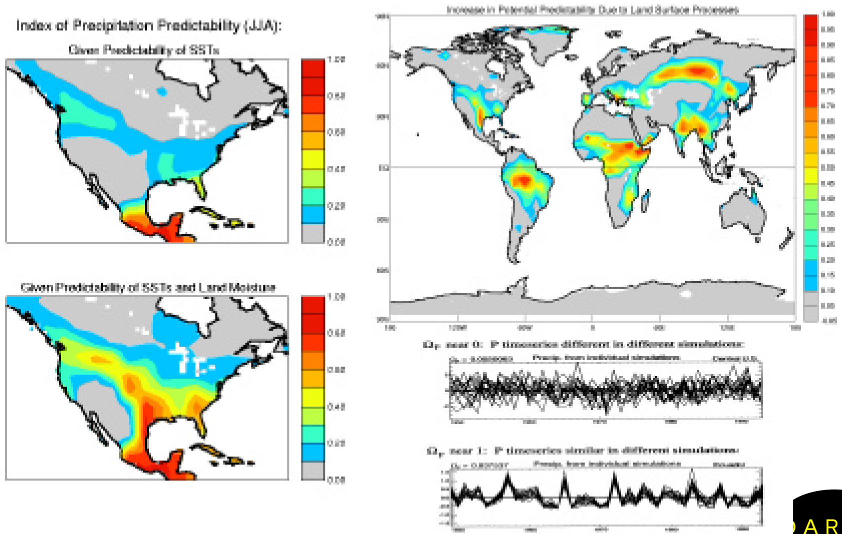
Land Initialization: Motivation

- Knowledge of soil moisture has a greater impact on the predictability of summertime precipitation over land at mid-latitudes than Sea Surface Temperature (SST).



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Conclusions

- Data assimilation should be essential part of “ground-segment” of all satellite missions.
- Aim should be to provide all data in “near real time”.
- Need to find optimal ways to assimilate data – efficient interfaces.
- Challenges: errors, resolution, data.

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