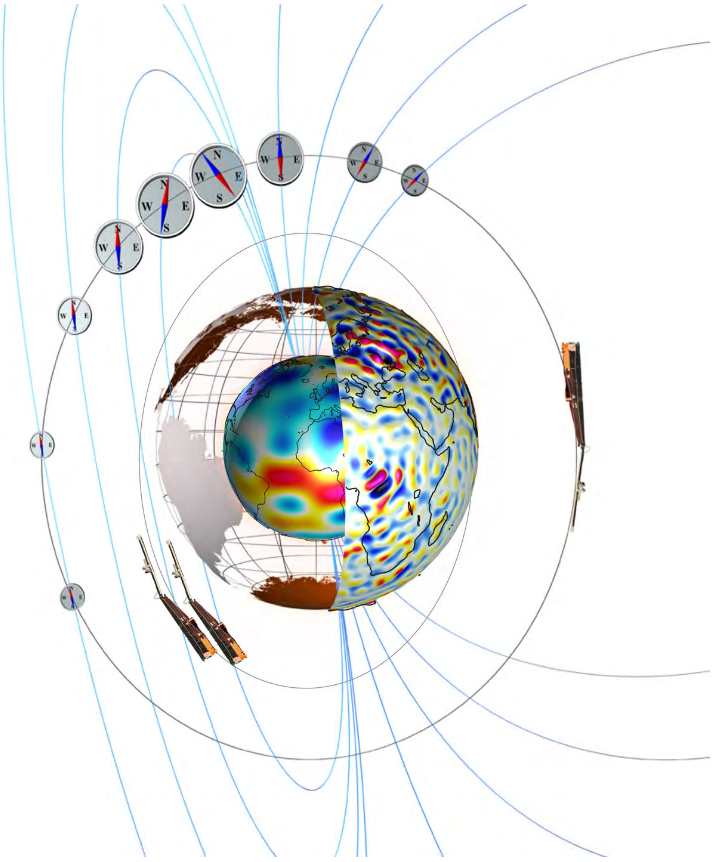
Data, Innovation, and Science Cluster
AMPS – Description of coefficient file format



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Preface

This document describes:
1: The format of the coefficient file, and how to use it
2: How the coefficients relate to ionospheric currents

1 The format of the coefficient file, and how to use it

The coefficient file, `'SW_OPER_MIO_SHA_2E_00000000T000000_99999999T999999_0101.txt'`, is an ascii file with fixed width columns. The file header contains the model name, time of production, reference to the paper where the model is described [Laundal et al., 2018], and various parameters used in making the model. The header lines have # as the first character. The last line of the header contains the column names. The first two columns are spherical harmonic degree and order, respectively. The remaining columns are described in more detail below.

The following lines of code read the coefficient file to a Python pandas `DataFrame`, with spherical harmonic wave numbers (`n, m`) as index, and with column names from the last line of the header.

```
import pandas as pd
coeff_fn = 'SW_OPER_MIO_SHA_2E_00000000T000000_99999999T999999_0101.txt'
names = ([x for x in open(coeff_fn).readlines() if x.startswith('#')][-1][1:]).strip().split(' ')
coeffs = pd.read_table(coeff_fn, skipinitialspace = True, comment = '#',
                       sep = ' ', names = names, index_col = [0, 1])
```

The resulting `pandas.DataFrame` will have 76 columns, named `'tor_c_'+'<param>'`, `'tor_s_'+'<param>'`, `'pol_c_'+'<param>'`, and `'pol_s_'+'<param>'` (adding two strings in Python appends the second to the first). Here, `'<param>'` refers to the 19 elements in the left column of Table 1.

The next step is to use the 14,402 coefficients in the file (excluding missing elements, see comment below) to make a reduced set of 758 coefficients that correspond to a specified set of external conditions. The following external conditions must be specified/chosen:

β	The dipole tilt angle [degrees]
v_x	Solar wind velocity in GSM/GSE x direction [km/s]
B_y	Interplanetary magnetic field GSM y component [nT]
B_z	Interplanetary magnetic field GSM z component [nT]
F10.7	F10.7 index [s.f.u.]

The first step to make the reduced set of coefficients is to multiply the columns of the coefficient table (`coeffs`) by a multiplier that is calculated from the external parameters. Table 1 shows, in the left column, the header suffix of the coefficient table columns that shall be multiplied by the number expressed in the middle column. Each of the multipliers will be applied to four columns.

The coefficient table elements, now scaled by the multipliers in Table 1, are used to construct the spherical harmonic coefficients in the representation of the ionospheric currents detailed in the next section. In that section we call the coefficients h_n^m, g_n^m, ψ_n^m, and η_n^m. They are produced by summing, across rows, the columns whose header prefixes coincide.

Using Python, the results are four `pandas.Series` objects, indexed by tuples `(n,m)`, which correspond to the spherical harmonic wave numbers (subscripts and superscripts in e.g., g_n^m). The four header prefixes correspond to the spherical harmonic coefficients as follows[1]:

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[1] 'pol' and 'tor' corresponds to poloidal and toroidal parts of the magnetic field, and _c and _s to cosine and sine terms, respectively, of the spherical harmonic expansions

Column header suffix	Multiplier	Comment
`'const'`	1	constant
`'sinca'`	$\sin\theta_c$	$\theta_c = \arctan2(B_y, B_z)$
`'cosca'`	$\cos\theta_c$	
`'epsilon'`	ϵ	$\epsilon = 10^{-3}\|v_x\|^{3/2}(B_y^2 + B_z^2)^{2/3}\sin^{8/3}(\theta_c/2)$
`'epsilon_sinca'`	$\epsilon\sin\theta_c$	
`'epsilon_cosca'`	$\epsilon\cos\theta_c$	
`'tilt'`	β	dipole tilt angle in degrees
`'tilt_sinca'`	$\beta\sin\theta_c$	
`'tilt_cosca'`	$\beta\cos\theta_c$	
`'tilt_epsilon'`	$\beta\epsilon$	
`'tilt_epsilon_sinca'`	$\beta\epsilon\sin\theta_c$	
`'tilt_epsilon_cosca'`	$\beta\epsilon\cos\theta_c$	
`'tau'`	τ	$\tau = 10^{-3}\|v_x\|^{3/2}(B_y^2 + B_z^2)^{2/3}\cos^{8/3}(\theta_c/2)$
`'tau_sinca'`	$\tau\sin\theta_c$	
`'tau_cosca'`	$\tau\cos\theta_c$	
`'tilt_tau'`	$\beta\tau$	
`'tilt_tau_sinca'`	$\beta\tau\sin\theta_c$	
`'tilt_tau_cosca'`	$\beta\tau\cos\theta_c$	
`'f107'`	F10.7 index	solar flux index in sfu

Table 1: Note that B_z and B_y should be given in nT, v_x in km/s, dipole tilt in degrees, and F10.7 in solar flux units. Also note the scaling of ϵ and τ by a factor of 10^{-3}. θ_c is the interplanetary magnetic field clock angle. ϵ is the Newell et al. [2007] coupling function.

`'pol_c'`: g_n^m
`'pol_s'`: h_n^m
`'tor_c'`: ψ_n^m
`'tor_s'`: η_n^m

Note: There are missing elements in the coefficient table, which are automatically filled in by `numpy.nan`'s using the recommended Python code for reading the file. Thus there will be nan's also in the resulting arrays of spherical harmonic coefficients. The reason for this is that not all coefficients are defined for all wave numbers. For example, h_n^0 and η_n^0 are all undefined, since they are coefficients of terms containing $\sin(m\phi)$, which are 0 for all ϕ if $m = 0$. The truncation level is also lower for the series that depend on h_n^m and g_n^m, so that these will be undefied for large n. It is safe to fill in 0's for the missing terms (but probably more efficient to skip entirely when calculating the sums described in the next section).

2 How the coefficients relate to ionospheric currents

In this section we describe how to use the spherical harmonic coefficients to calculate ionospheric currents. The description is based on Laundal et al. [2016, 2018]. We refer to these papers for more details about the background for the formulas presented below. The notation used here is similar to that used in these papers, including the spherical harmonic coefficients h_n^m, g_n^m, ψ_n^m, and η_n^m.

In each expression, we sum over all pairs of spherical harmonic wavenumbers (n, m) for which the coefficients are defined. This is determined by the chosen truncation level. This is $N = 45, M = 3$ for h_n^m and g_n^m, and $N = 65, M = 3$ for ψ_n^m and η_n^m. We repeat that h_n^0 and η_n^0 are always undefined, since $\sin(m\phi) = 0$ for all ϕ if $m = 0$.

The spatial coordinates are quasi-dipole or modified apex magnetic latitude, λ_q or λ_m, repectively, and magnetic local time. The currents are calculated at a fixed height, h_R, normally set to 110 km, although it does not have to be. At h_R, $\lambda_q = \lambda_m$, so that we skip the subscripts in the following. The magnetic co-latitude $\theta = 90° - \lambda$. The magnetic local time, ϕ, is defined as recommended in Laundal and Richmond [2017].

The expressions below also depend on the Schmidt semi-normalized associated Legendre functions, $P_n^m(\theta)$. The unit of the spherical harmonic coefficients are nT. The Earth radius used in the expressions below are $R_E = 6371.2$ km. μ_0 is the permeability constant, defined to be $4\pi 10^{-7}$ Tm/A.

2.1 Vertical / field-aligned current

The upward current at (λ, ϕ), where λ is magnetic latitude $(90° - \theta)$ and ϕ magnetic local time, calculated at a height h_R, is

$$\boxed{J_u(\lambda, \phi) = -\frac{10^{-6}}{\mu_0(R_E + h_R)} \sum_{n,m} n(n+1) P_n^m(\theta) \left[\psi_n^m \cos m\phi + \eta_n^m \sin m\phi\right]}$$

The unit is $\mu\text{A/m}^2$. Explanation: h_R and R_E are given in km and the coefficients in nT. We get: $\text{A}/(\text{T} \cdot \text{m})/\ \text{km} \cdot \text{nT} = 10^{-12}\text{A/m}^2$, or picoAmps per m^2. To convert to $\mu\text{A/m}^2$, we multiply by 10^{-6}, hence the factor in the expression above.

2.2 Horizontal current

We denote the horizontal current sheet density as $\mathbf{J}$. This quantity can be interpreted as the height-integrated current density projected at a height h_R. The current can be written as a sum of divergence-free (df) and curl-free (cf) parts (Helmholtz decomposition):

$$\mathbf{J} = \mathbf{J}_{df} + \mathbf{J}_{cf} = \mathbf{k} \times \nabla\Psi + \nabla\alpha, \tag{1}$$

where we have written $\mathbf{J}_{df}$ in terms of a scalar field Ψ, and $\mathbf{J}_{cf}$ in terms of a scalar field α. $\mathbf{k}$ is an upward unit vector.

The scalar for the divergence-free part, Ψ, is:

$$\boxed{\Psi(\lambda, \phi) = -\frac{R_E}{\mu_0} \sum_{n,m} \frac{2n+1}{n} \left(\frac{R_E}{R_E + h_R}\right)^{n+1} P_n^m(\theta)\left[g_n^m \cos m\phi + h_n^m \sin m\phi\right]}$$

The scalar for the curl-free part, α, is:

$$\boxed{\alpha(\lambda, \phi) = -\frac{R_E + h_R}{\mu_0} \sum_{n,m} P_n^m(\theta)\left[\psi_n^m \cos m\phi + \eta_n^m \sin m\phi\right]}$$

In both cases the unit is μA.

Notice that α depends only on ψ_n^m, and η_n^m, the same coefficients as J_u. This is because α is a solution to $\nabla^2\alpha = -J_u$. Ψ is independent of J_u, and depends only on g_n^m and h_n^m.

The expressions for Ψ and α can be inserted in Equation 1 to get the current sheet density (using spherical coordinate versions of the differential operators). The current sheet density will have dimension current per length. It quantifies how much (sheet) current flows across a line of unit length, which is perpendicular to $\mathbf{J}$.

Annex: Product definition table

	MIO_SHA_2E
Definition	An empirical model of the Average Magnetic field and Polar current System (AMPS). See forward code at https://doi.org/10.5281/zenodo.1182931, and science paper at https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2018JA025387.
Input Data	Swarm magnetic field (MAG-L) measurements, CHAMP magnetic field measurements, model predictions of core, lithospheric and large-scale magnetospheric (including Earth-induced) contributions, taken by the CHAOS geomagnetic field model, OMNI solar wind measurements, F10.7 index
Input Time Span	Entire Swarm mission + entire CHAMP mission
Spatial representation	Spherical harmonic expansion in QD/modified apex coordinates, with degree up to 45/65 for poloidal/toroidal magnetic field parts. Spherical harmonic order (in MLT) up to 3.
Time representation	Time dependence contained in parametrization in terms of external parameters: 20 min average solar wind speed and interplanetary magnetic field, dipole tilt angle, and F10.7 index
Units	Magnetic field in nT, horizontal sheet current density in mA/m, vertical current density in microA/m2
Resolution	0.1 nT, 1 mA/m, 0.01 microA/m2
Uncertainty	See https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2018JA025387
Quality Indicator	N/A
Data Volume	218 kB
Data Format	ascii
Output Data	Spherical harmonic coefficient
Output Time Span	Model is based on data from entire CHAMP and Swarm missions, but it is in principle valid for any time when solar wind data was available.
Update Rate	Yearly (to be confirmed)
Latency	2 months, due to update rate of OMNI data and computation time.
Notes	

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